

IOWA STATE UNIVERSITY

ECpE Department

Distribution System Resilience: Hardening, Preparation and Restoration

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Outline



- Introduction to Resilience



- Pre-Event Preparation



- Post-Event Outage Management and Service Restoration



- Resilience-Oriented Long-Term Planning

History of Resilience

- The word resilience is found from the year 1430 in late medieval and early modern French as a juridical term for contract termination and for the restoration of the original legal situation.
- In 1818, Tredgold used resilience to explain why some types of wood were able to accommodate sudden and severe loads without breaking.
- In 1856, Robert Mallet further developed this concept of resilience as a means of measuring and comparing the strength of materials used in construction.
- In 1973, Crawford Holling introduced the concept of resilience to ecology and the environment. He defined it as a measure of the persistence of systems and their ability to absorb change and disturbance.

History of Resilience (cont.)

- In 1977, Amory Lovins adapted Holling's resilience concept to energy systems in his article "Resilience In Energy Strategy."
- In 2000, Neil Adger introduced resilience to social science and defined it as the ability of communities to withstand external shocks to their social infrastructure.
- In 2009, the National Infrastructure Advisory Council defined critical infrastructure resilience as:
 "...the ability to reduce the magnitude and/or duration of disruptive events. The effectiveness of a resilient infrastructure or enterprise depends upon its ability to anticipate, absorb, adapt to, and/or rapidly recover from a potentially disruptive event."

Definitions

Resilience The ability to prepare for and adapt to changing conditions and withstand and recover rapidly from disruptions.

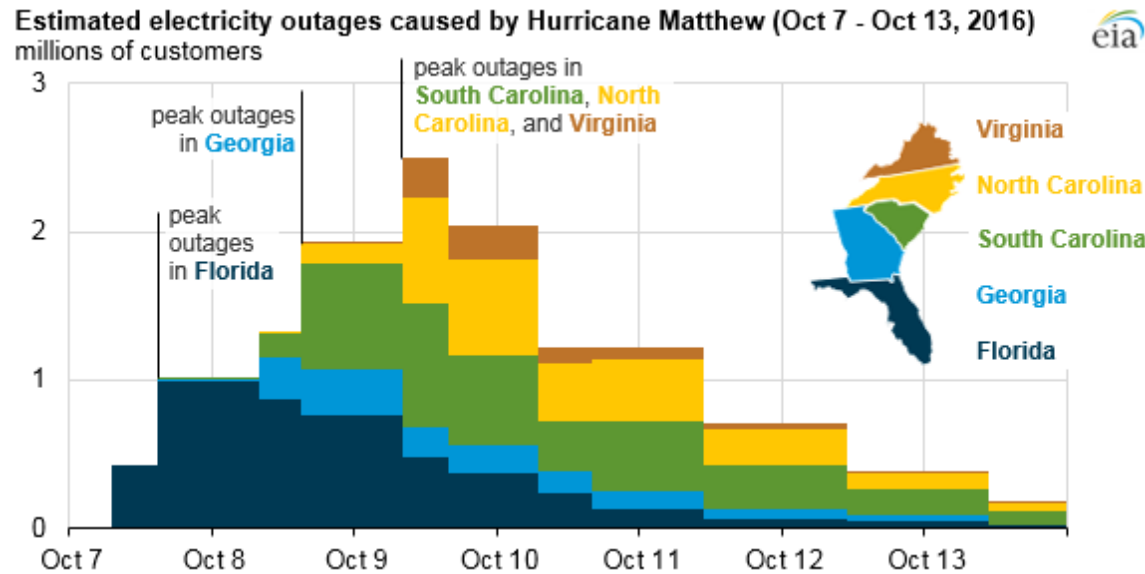
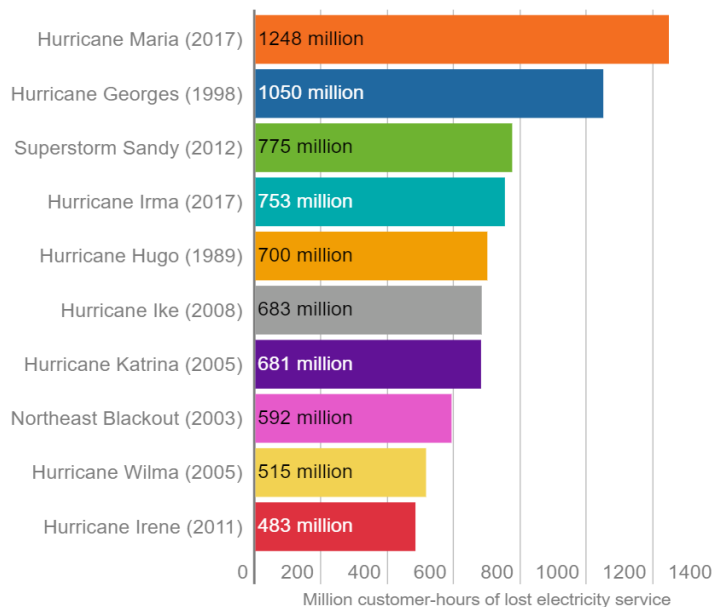
Reliability The ability of the system to satisfy the customer demand within accepted standards and in the amount desired.

	Resilience	Reliability
Events Considered	Low Probability, High Consequence Hazards	High Probability, Low Consequence Hazards
Risk-based?	Yes	No
Binary or continuous?	Resilience is considered a continuum, confidence is specified	Operationally, the system is reliable or not. Confidence is unspecified
Measurement focus	Focus is on measuring impact to humans	Focus is on measuring the impact to the system

Source: (Vugrin 2017)

Extreme Weather and Power Grid

- Extreme weather events constantly threaten and damage electric power systems.
- Overhead distribution systems are vulnerable to severe weather events such as hurricanes, wind storms, heavy rain, lightning, ice, freezing rain, and snow.
- Recent years have seen an increase of weather events and outages.



Reliability Metrics

- SAIFI - System Average Interruption Frequency Index

Average frequency of sustained interruptions per customer:

$$\frac{\text{Number of interrupted customers}}{\text{Total number of customers}}$$

- SAIDI - System Average Interruption Duration Index

Customer minutes of interruption or customer hours :

$$\frac{\text{Sum of all customer interruption durations}}{\text{Total number of customers}}$$

- CAIDI - Customer Average Interruption Duration Index

Average time needed to restore service to the average customer: $\frac{\text{SAIDI}}{\text{SAIFI}}$

- These metrics reflect the system reliability, not resilience
- A system can be reliable but not resilient

Reliability Metrics (Cont.)

- Why reliability metrics cannot be directly applied to resilience?
 - 1) Undervalue the impact of large-scale events and focus on normal operating conditions;
 - 2) High standard deviation.
- Many utilities exclude major events from SAIFI and SAIDI.
- There is a need to design new metrics for resilience.

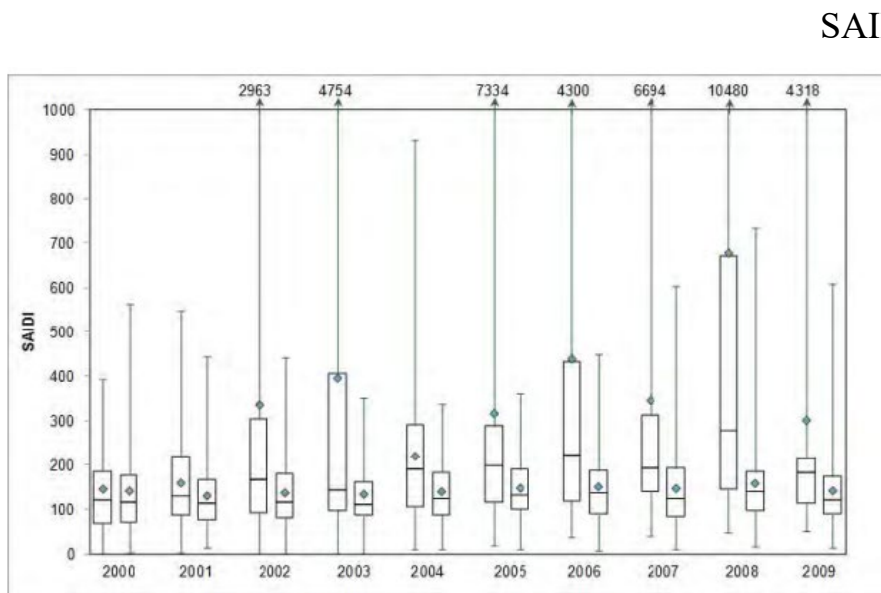


Figure . Box-Plot of SAIDI by Year with and without Major Events

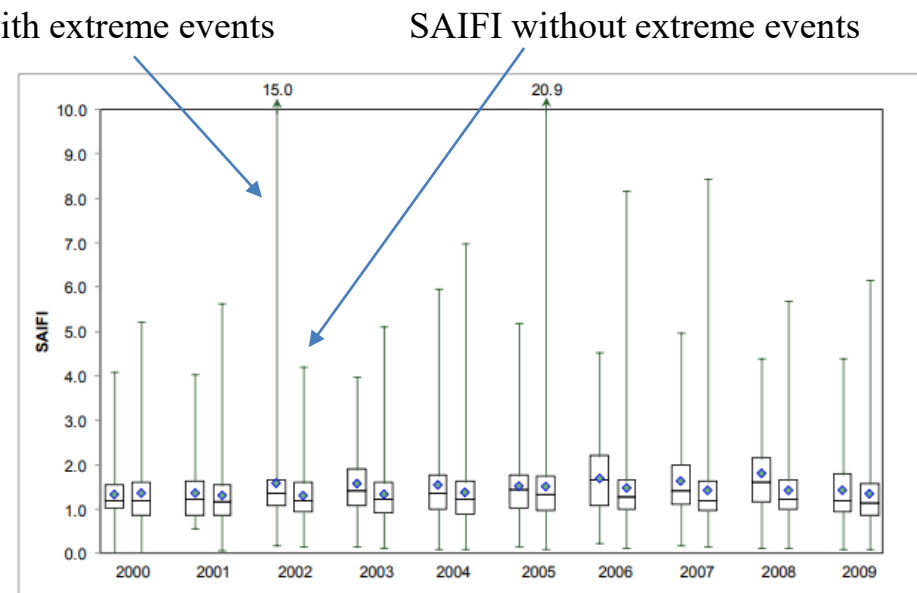
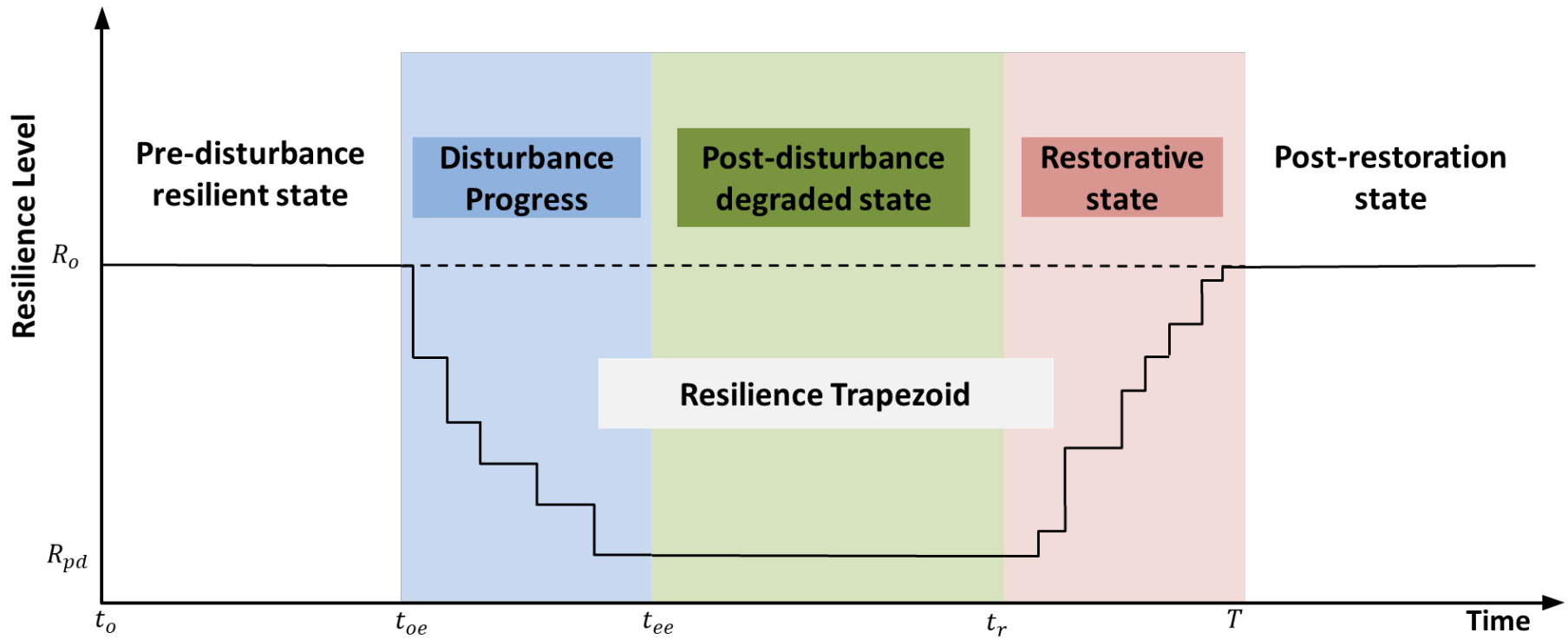


Figure . Box-Plot of SAIFI by Year with and without Major Events

Resilience Curve

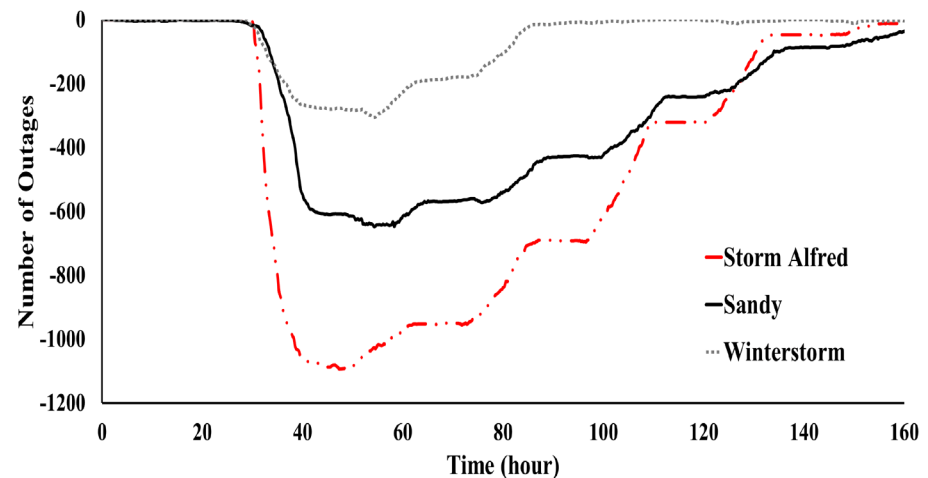
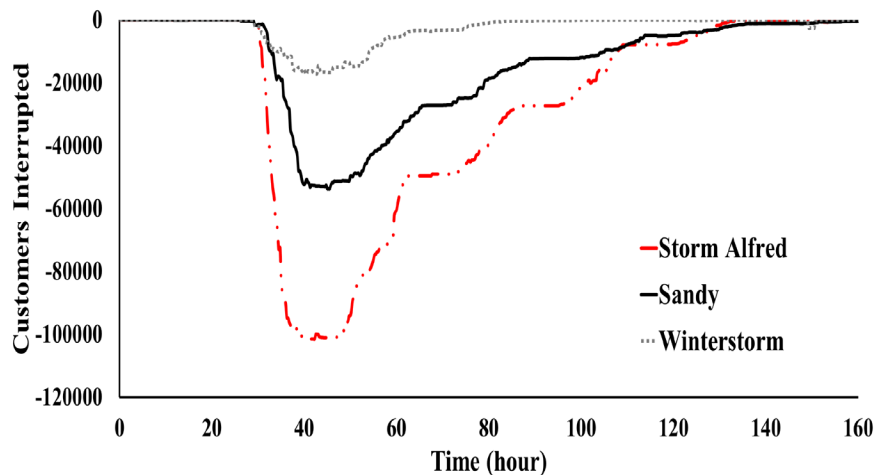


Time Sequence	Event hits the network	End of event	Restoration is initiated	End of restoration	
Type of Actions	Preventive	Corrective	Emergency Coordination	Restorative	Adaptive

(Panteli 2017)

Resilience Curves-Real Data

- The figures show the number of interrupted customers and outages for three different events
 - Storm Alfred (October 2011)
 - Hurricane Sandy (October 2012)
 - Winter storm (November 2014)
- Storm Alfred occurred two months after Hurricane Irene

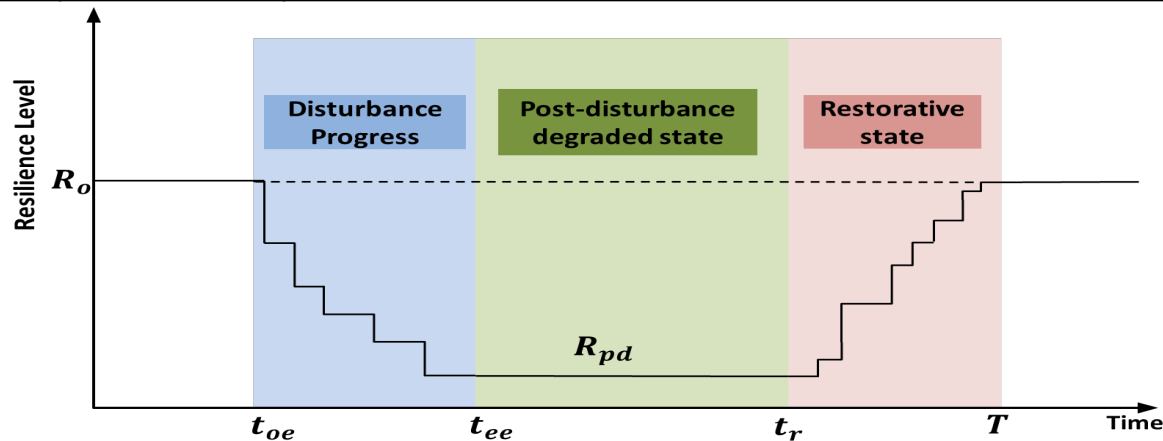


Resilience Metrics (1/3)

Metric	Equation	Description
Storm Average Interruption Frequency Index (STAIFI)	$\frac{\text{Total Number of Customers Interrupted}}{\text{Total Number of Customers Served}}$	<ul style="list-style-type: none"> • STAIFI and STAIIDI exhibit too much uncertainty because of their high standard deviation.
Storm Average Interruption Duration Index (STAIIDI)	$\frac{\text{Total Customer Storm Interruption Minutes}}{\text{Total Number of Customers Served}}$	<ul style="list-style-type: none"> • The metrics are static and do not represent the dynamic evolution of damage and recovery processes. • Insufficient representation of the physical aspect of grids.
Estimated Time of Restoration (ETR)	Time of Outage + Estimated Recovery Time	<ul style="list-style-type: none"> • Difficult to estimate due to the uncertainties in the recovery process. • Fails to provide a clear indication of the network's ability to withstand weather events.

Resilience Metrics (2/3)

Metric	Equation	Description
Speed of degradation	$\frac{R_{pd} - R_0}{t_{ee} - t_{oe}}$	<ul style="list-style-type: none"> This metric indicates how fast the resilience drops after an extreme event. It can be used to measure the network's ability to withstand the event, but not for recovery.
Amount of degradation	$R_0 - R_{pd}$	<ul style="list-style-type: none"> The metric measures the initial impact of the extreme event.
Duration of the post-disturbance degraded state	$t_r - t_{ee}$	<ul style="list-style-type: none"> Indicates the quality of the initial immediate response after the event. This metric highly depends on the fault location, isolation and service restoration (FLISR) technologies being used.
Speed of network recovery	$\frac{R_0 - R_{pd}}{T - t_r}$	<ul style="list-style-type: none"> The metric measures the quality of the response from the utility. It includes the speed of damage assessment, repair process and crew management, and power restoration operation.
Area of the resilience trapezoid	$\int_{t_{oe}}^T R(t) dt$	<ul style="list-style-type: none"> This metric gives an overall indicator of the system performance.



Resilience Metrics (3/3)

- Measure resilience at the network-level involving both infrastructure and services (Wei 2013)
- Combine the **infrastructure** and **service** resilience metrics (Ji 2017)

$$R(t) = 1 - \frac{1}{C_0} E\{C(t; d)\}$$

d : a threshold on tolerable delays for recovery

C_0 : is a normalization factor

➤ Where:

$$E\{C(t; d)\} = \int_0^{t-d} E_{S(v)} \left\{ \lambda_i^f(v|S(v)) E\{G_i(v, t)|S(v)\} \right\} dv$$

Expected state of the system $S(v)$

Failure rate of the infrastructure

Expected disruption cost

- ❖ This metric does not include weather variables. An open issue is how to derive resilience metrics combining weather with the infrastructure and services.

Resilience Enhancement

Phases	Actions
Long-term planning	<ul style="list-style-type: none">• Infrastructure hardening• Vegetation management• Adding distributed energy resources (DER)• Implementing smart grid technologies<ul style="list-style-type: none">• Automated switching devices and sensors• Smart meters for situational awareness
Short-term pre-event preparation	<ul style="list-style-type: none">• Weather forecast and damage prediction• Pre-position crews• Pre-allocate equipment and fuels• Pre-position mobile energy sources
Post-event restoration	<ul style="list-style-type: none">• Automatic fault isolation and service restoration• Improved damaged assessment<ul style="list-style-type: none">• Damage location prediction• Smart meters• Drones• Optimizing repair scheduling and crew routing• Dynamic network reconfiguration• Use of DERs, demand response, and microgrids for restoration

Our Research on Resilience

- Resilience: The ability to prepare for and adapt and recover rapidly from disruptions.

- **A tri-stage robust optimization model**
- **A two-stage stochastic optimization model**
- S. Ma, S. Li, Z. Wang, F. Qiu, Resilience-Oriented Distribution System Design with Decision-Dependent Uncertainty, *IEEE Trans. Power Syst.*, 2018.
- S. Ma, L. Su, Z. Wang, F. Qiu, Resilience Enhancement of Distribution Grids Against Extreme Weather Events, *IEEE Trans. Power Syst.*, 2018.
- S. Ma, B. Chen, Z. Wang, Resilience enhancement strategy for distribution systems under extreme weather events, *IEEE Trans. Smart Grid*, 2016.

- **Co-optimize distribution grid operation and crew repair**
- A. Arif, Z. Wang, J. Wang, C. Chen, "Repair and resource scheduling in unbalanced distribution systems using neighborhood search," *IEEE Trans. Smart Grid*, accepted, 2019.
- A. Arif, S. Ma, Z. Wang, J. Wang, S. M. Ryan, C. Chen, "Optimizing service restoration in distribution systems with uncertain repair time and demand," *IEEE Trans. Power Syst.*, vol. 33, no. 6, pp. 6828-6838, Nov. 2018.
- A. Arif, Z. Wang, J. Wang, C. Chen, "Power distribution system outage management with co-optimization of repairs, reconfiguration, and DG dispatch," *IEEE Trans. Smart Grid*, vol. 9, no. 5, pp. 4109-4118, Sept. 2018.

Planning

Preparation

Damage Assessment

Repair & Restoration

- Tree trimming
- DERs
- Automatic Switches
- Hardening
- Microgrids

- A. Arif, Z. Wang, C. Chen, B. Chen, A stochastic multi-commodity logistic model for disaster preparation in distribution systems, *IEEE Trans. Smart Grid*, accepted, 2019.

- Weather forecasting
- Outage modelling and prediction
- Crew and equipment allocation

- Fault location
- UAVs
- Repair time prediction

- A. Arif, Z. Wang, "Distribution network outage data analysis and repair time prediction using deep learning," *IEEE Int Conf. Probabilistic Methods Appl. Power Syst.*, Boise, ID, 2018.

- Fault isolation and service restoration
- Dispatch repair crews and repair scheduling
- Microgrid formation

Motivation

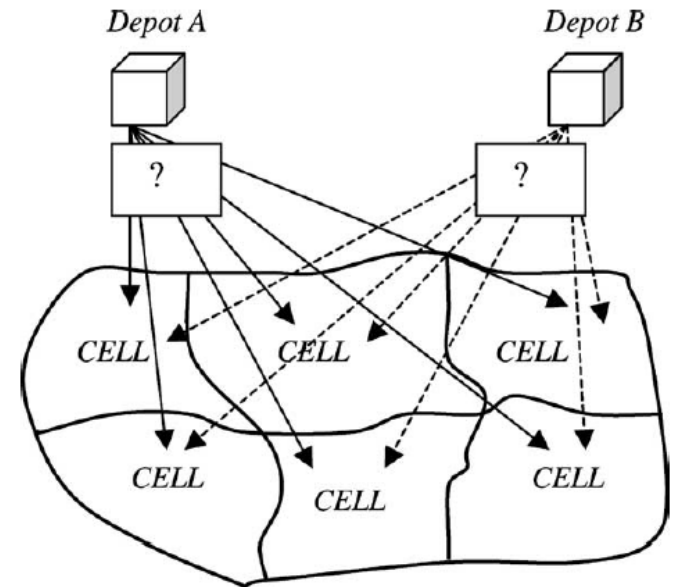
- Severe power outages caused by extreme weather events
 - Hurricane Irene (2011): 6.69 million customers
 - Hurricane Sandy (2012): : 8.66 million customers
 - Hurricane Irma (2017): 15 million customers
 - Cost of weather-related outages: \$25 to \$70 billion annually in U.S.
- The energy infrastructure is aging, inefficient, and highly vulnerable to extreme weather
- We need a resilient system that can withstand the extreme events and recover quickly after the event



Part I Pre-event Preparation

Review – Disaster Preparation

- Few studies focused on disaster preparation in the context of power system and its infrastructure
- The previous work approached the preparation stage by dividing the electric network into different areas, with each area having a specific demand



(Wang 2004)

Ref.	Application	Method
Wang 2004	Find optimal number of depots and their locations around the power network	MILP
Coffrin 2011	Determine the number of resources to stockpile before a disaster in order to repair the power network	SMIP
Khomami 2018	Preposition repair crews before a disaster near expected damaged components	Heuristic

Review – Distribution System Restoration

- **Reconfiguration:** optimal reconfiguration of the distribution network with the objective of maximizing the served loads
- **Reconfiguration and DG dispatch:** optimal reconfiguration of the distribution network and DG operation
- **Microgrids:** optimal operation of microgrids for service restoration
- **Repair Scheduling:** repair scheduling of distribution systems' assets without considering network operations

Method	Model/Algorithm			
	MILP	Stochastic/Robust	Agent-based	Heuristic
Reconfiguration	Butler 2018	Lee 2015	Solanki 2007	Kumar 2008
Reconfiguration+DGs	López 2018	Chen 2016	Zidan 2012	Drayer 2018
Microgrids	Wang 2016	Wang 2015	Zhao 2018	Hu 2017
Repair Scheduling	Golla 2017	Xu 2007		Johns 1994

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MILP: Mixed integer linear Program

Review: Repair and Restoration

- ❖ **How do utilities schedule the repairs?**

- ❖ Define priorities for the damaged components → dispatch the crews according to the priorities

- ❖ 2-Step approach for **transmission** systems (Pascal Van Hentenryck and Carlton Coffrin 2015):

1. Restoration Ordering Problem: assume only one component can be repaired at each time step

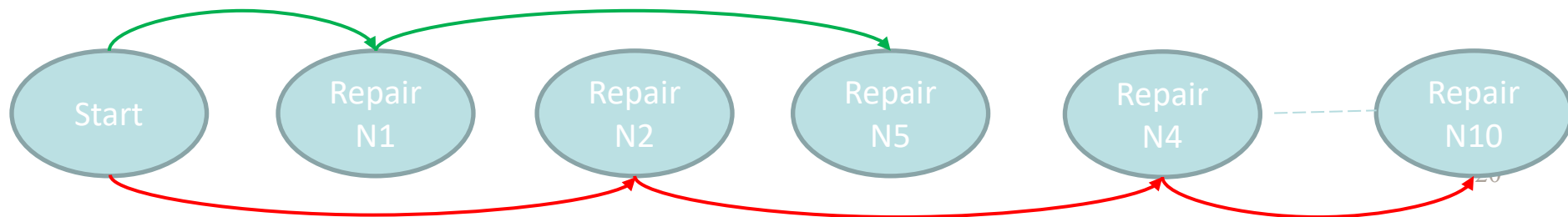
- ❖ Solved using MILP



2. Routing: solve a routing problem with precedence constraints

- ❖ Solved using Constraint Programming

- ❖ Precedence constraint



Problem Statement

What is missing?

- An optimization strategy for disaster preparation that selects staging areas and allocates crews and equipment while considering the system's components
- A co-optimization method that jointly optimizes crew routing and distribution system operation
- Solution algorithms for solving these difficult problems

Pre-event preparation

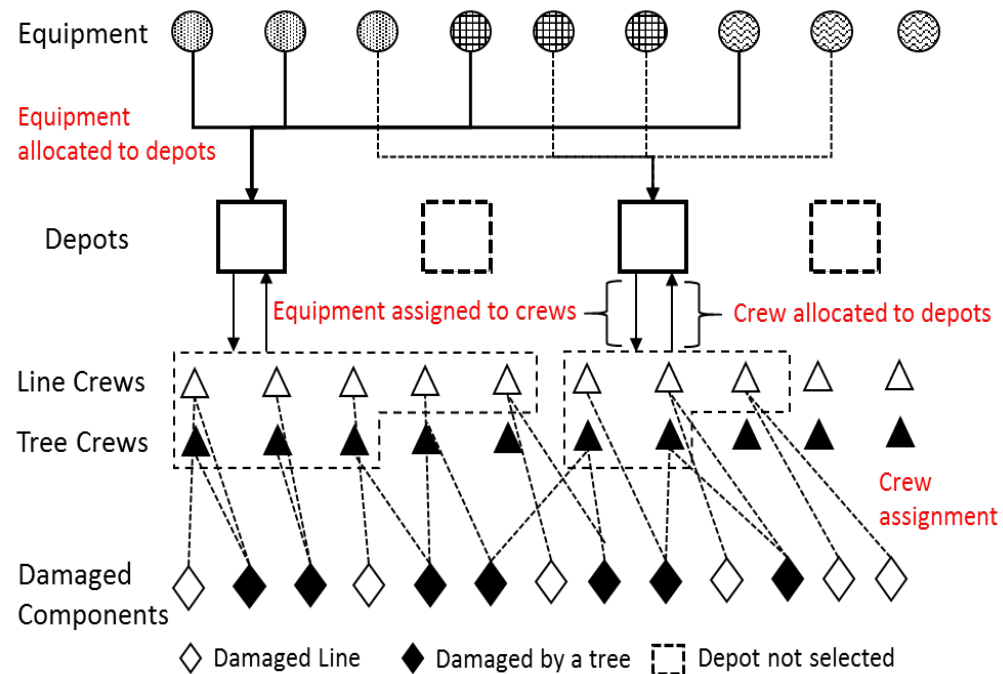
- Choose staging locations
- Mobilize available crews and request assistance if necessary
- Obtain and allocate equipment

Post-event repair and restoration

- Coordinate tree and line crews
- Manage equipment
- Isolate damaged components
- Operate the distribution system

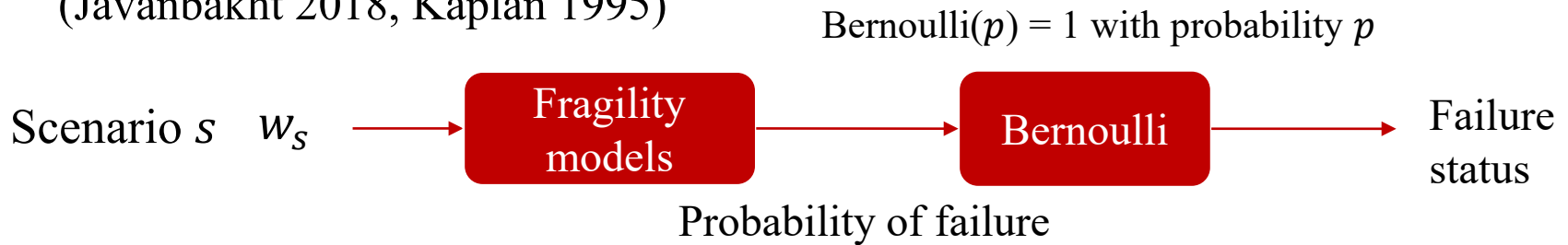
Summary

- Develop a two-stage stochastic program
- Use fragility models to generate scenarios
- Uncertainties: damaged components, equipment, and repair times
- Objective:
 - Minimize preparation costs and penalty over unmet demand and late repairs
- First-stage:
 - Depot selection
 - Crew and equipment allocation
- Second-stage:
 - Assign crews to damaged components

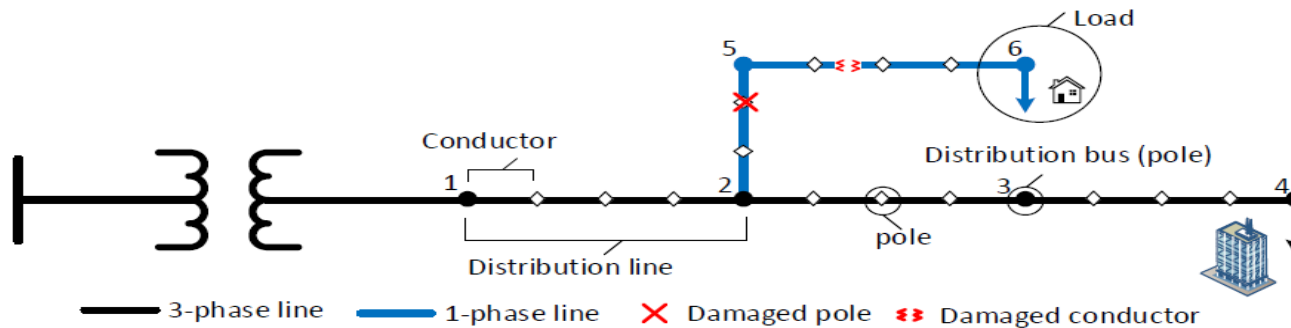


Scenario Generation (1/2)

- Assuming a hurricane is forecasted
- We generate wind speeds using lognormal distribution and hurricane model (Javanbakht 2018, Kaplan 1995)



- Fragility models to (Ouyang 2014):
 - Calculate the probability of failure of each pole
 - Calculate probability of failure of each conductor
 - Probability of wind induced damage
 - Probability of damage due to fallen trees



Scenario Generation (2/2)

- Calculate required equipment (poles, transformers, conductors)
- Estimate the repair times using normal distributions (Ouyang 2014)
- Identify critical components
 - Solve a MILP to identify minimum number of lines to repair
 - Minimize number of lines to be repaired while serving all critical loads
 - Status of the line: u_k
 - Status of the load: y_i

$$\min \sum_{k \in \Omega_{DL}(s)} u_k$$

$$\text{subject to } y_i = 1, \forall i \in \Omega_{CD}$$

subject to power operation constraints

Objective

- First-stage objective: minimize the costs of equipment transportation, ordering equipment and external crews, and staging depots

$$\min \sum_{\forall d,e,\tau} \mathcal{P}_{d,e,\tau}^{TE} E_{d,e,\tau} + \sum_{\forall d,\tau} \mathcal{P}_{\tau}^{EI} EI_{d,\tau} + \sum_{\forall d} (\mathcal{P}^{EC} (LI_d + \mathcal{T}I_d) + \mathcal{P}_d^D \nu_d)$$

- Second-stage objective:
 - Minimize the costs associated with the crews. The costs of crews include labor, food, and accommodation
 - Minimize penalty costs of unmet equipment demand and time it takes to repair all components

$$\min \sum_{\forall s} \Pr(s) \left(\sum_{\forall c} \mathcal{P}_c^H H_{c,s} + \sum_{\forall d,\tau} \mathcal{P}_{\tau}^{LF} \mathcal{E}_{d,\tau,s} + \mathcal{P}^R (\mathcal{L}_s^T + \mathcal{L}_s^L) \right)$$

Constraints

First-stage constraints

- Select depots
- Transfer existing equipment/crews between depots
- Acquire new equipment/crews
- Depot capacity constraint

Second-stage constraints

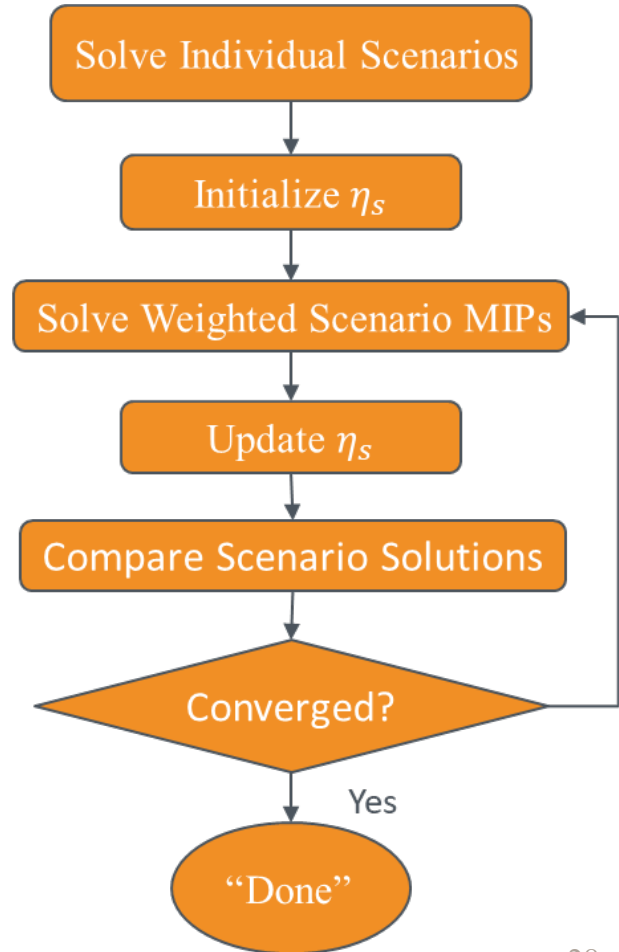
- Crews are assigned to repair damaged the components
- The assignment is constrained by the distance
- Calculate working hours
- Assign equipment to the crews
- We must have enough equipment for critical components
- Calculate unmet equipment demand

Solution Methods

- The Extensive Form (EF)
 - Write down the full variable and constraint set for all scenarios
 - Attempt to solve with a commercial MIP solver
 - Best solution, but often does not work due to memory or time limits
- Progressive hedging
 - Scenario-based decomposition
 - Pros: parallelizable
 - Cons: heuristic

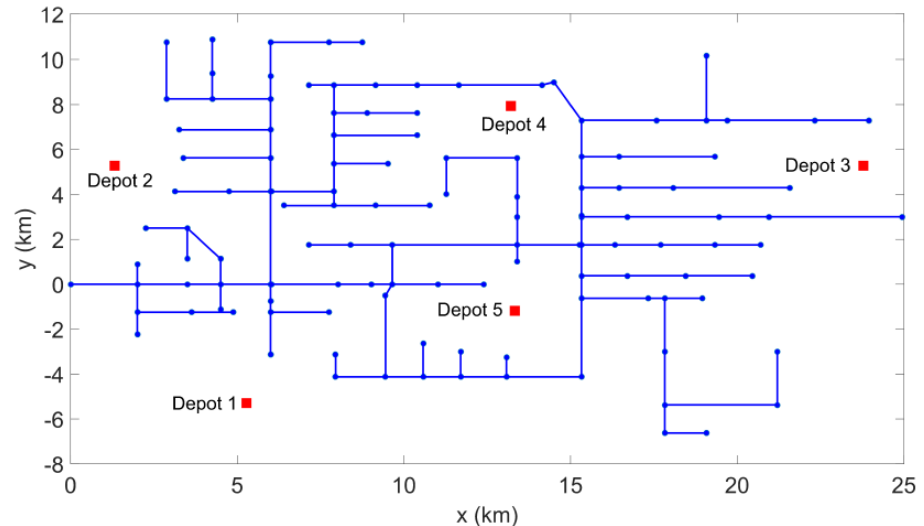
Solution Algorithm - Progressive Hedging

- Algorithm:
 - Solve each scenario independently
 - Find the average first-stage solution $\bar{x} = \sum_{\forall s} \Pr(s) x_s$
 - Calculate penalty factor $\eta_s = \rho(x_s - \bar{x})$
 - Augment the penalty factor to the stochastic model and solve
 - If $\sum_{\forall s} \Pr(s) \|x_s - \bar{x}\| > \epsilon$ go to 2
- The algorithm terminates once all first-stage decisions x_s converge to a common \bar{x}
- The PH algorithm may experience slow convergence
- We fix some of the first-stage variables (depot selection and crew allocation) if they converge to the same values after some number of iterations



Results (1/2)

- IEEE 123-bus system
- Proposed method (SCRAP) is compared with:
 - Deterministic allocation (DA)
 - Robust stochastic optimization method (RSO) (Bozorgi-Amiri 2013)
- Main difference is in the number of equipment acquired
- The deterministic solution did not consider some of the extreme cases
- RSO favors a solution that would perform better with worst-case scenarios



PRE-EVENT PREPARATION RESULTS

		SCRAP		DA		RSO	
Staged Depots		1	4	1	4	1	4
Line Crews		6	4	6	4	6	4
Tree Crews		2	1	2	1	2	1
Equipmant	1	10	6	10	0	15	8
	2	16	13	13	6	26	15
	3	3	0	3	0	3	0
	4	6	2	7	1	6	3
	5	3.8 km	2 km	2.5 km	1.5 km	5 km	3 km
Costs		\$146,766		\$117,443		\$183,371	

Results (2/2)

- The wait-and-see (WS) solution is calculated to provide a lower bound
- We calculate the objective value of the stochastic model for each method by using the first-stage decisions of the different methods

Method	Objective Value	Computation Time
WS	\$513,170	N/A
SCRAP-EF	\$549,554	300 min
SCRAP-PH	\$551,585	106 min
RSO	\$608,683	335 min
ED	\$714,602	2 min

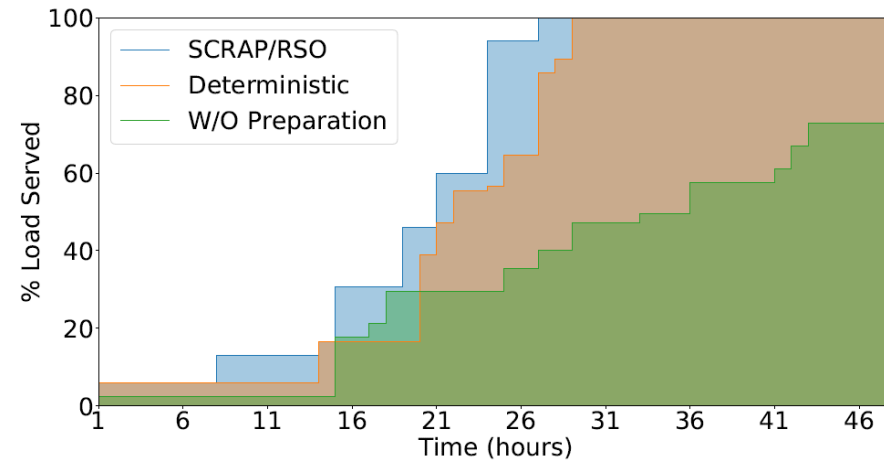
Restoration Phase

- To assess the devised preparation plan, we solve the repair and restoration problem with and without preparation
- A new random damage scenario is generated on the IEEE 123-bus system
- The stochastic and robust models have enough equipment, however, RSO has a large surplus

REPAIR AND RESTORATION PERFORMANCE AFTER THE EVENT

Preparation	Equipment	Load served (kWh)
SCRAP	{+3,+11,+3,+6,+0.32 km}	80,136 kWh
RSO	{+10,+23,+3,+7,+3.7 km}	80,136 kWh
DA	{-3,+1,+3,+6,-0.34 km}	77,448 kWh
W/O Preparation	{-3,-3,+3,+6,-0.34 km}	46,667 kWh

“-”: shortage; “+”: surplus; the load served is for the first 48 hours
 Equipment: {Poles for 3-phase lines, Poles for single-phase lines, 3-phase transformers, single-phase transformers, conductor}

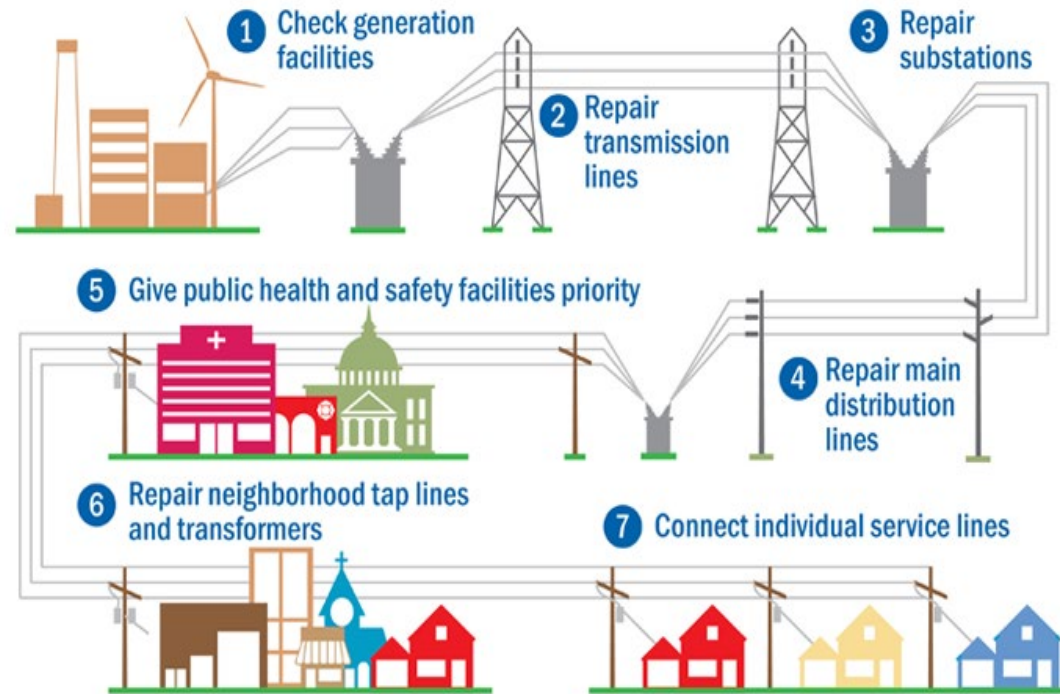


Part II Post-event Outage Management and Service Restoration

Distribution System Outage Management

Distribution system outage management involves utility procedures and computer-based tools to efficiently and effectively:

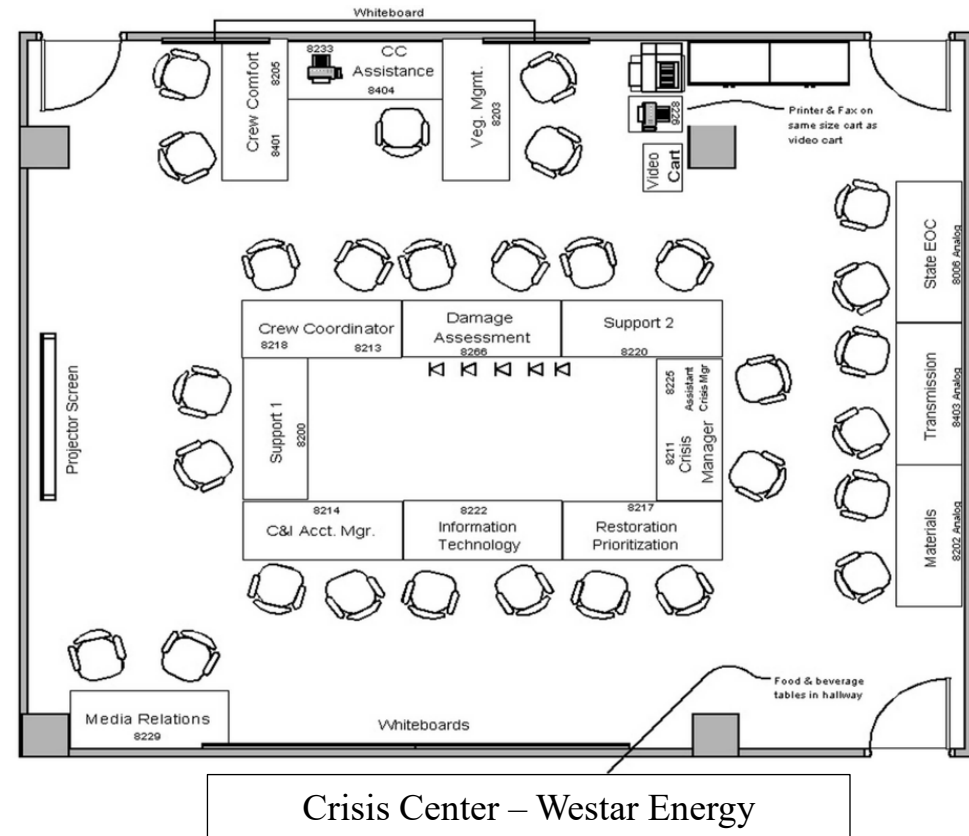
- Predict and prepare for outages
- Detect and locate outages
- Dispatch crews and manage equipment
- Restore the distribution system
 - isolate faults and restore the healthy sections by reconfiguring the network
- Provide feedback to affected customers



Source: <https://www.nppd.com/outages/restoring-power/>

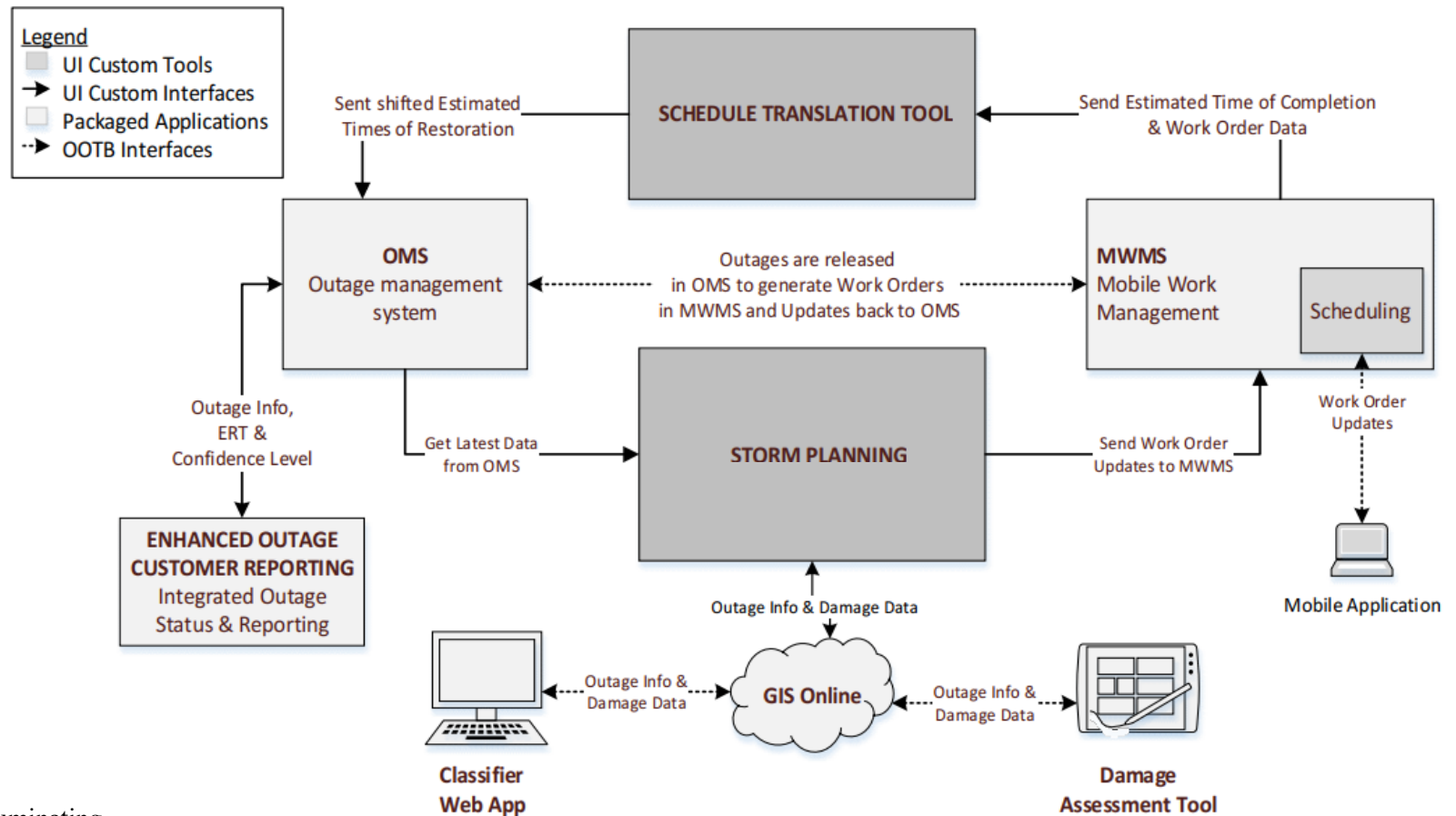
Utility Practices (1/3)

- Preparation
 - Crews and staff on alert
 - Request assistance
 - Pre-storm allocation of crews and resources
- Outage Management System
 - Data from customer calls, SCADA, AMI, etc are collected
 - Determines the likely location of the trouble
- Damage assessment process
 - Damage assessors navigate to the outage locations
 - Record damage data
- Prioritizing restoration activities
 - Hazards → critical customers (e.g., hospitals) → prioritize by number of customers
- Crew Scheduling
 - Crews are assigned to different areas for large systems
 - Schedule in sequence of priority



Source: <https://www.westarenergy.com/>

Utility Practices (2/3)



System Architecture Diagram

UI: United Illuminating

ETC: Estimated Time of Completion

ERT: Estimated Restoration Times

MWMS: Mobile Work Management System



Utility Practices (3/3)

Challenges

- Distribution systems are becoming more complex with new devices and systems. DGs and automatic switches can greatly decrease the restoration time if operated effectively.
- Managing crews, equipment, and the operation of the network is a demanding task. After an extreme event, a sudden influx of crews can overwhelm operators and storm planners.
- The recovery operation problem and repair scheduling are interdependent.
- Currently, crews are scheduled based on a priority list. If the priorities are not well defined, the schedule will not be efficient.

Improvements

- Development of advanced optimization methods to jointly optimize the recovery operation and logistic problems. An optimization process can help the operator in making critical and more informed decisions after outages.
- Design solution algorithms for the co-optimization problem to obtain a quick₃₆ and efficient solution

Distribution System Restoration

- **Reconfiguration:** optimal reconfiguration of the distribution network with the objective of maximizing the served loads.
- **Reconfiguration and DG dispatch:** optimal reconfiguration of the distribution network and DG operation.
- **Networked Microgrids:** optimal operation of interconnected individual microgrids with defined boundaries.
- **Microgrid formation:** optimal operation of microgrids with dynamic boundaries.
- **Repair Scheduling:** repair scheduling of distribution systems' assets without considering network operations.

Method	Model/Algorithm				
	MILP	Stochastic/Robust	Agent-based	Heuristic	Other
Reconfiguration	[3]-[5]	[6]-[7]	[8]-[10]	[12]-[14]	[15]-[17]
Reconfiguration+DGs	[18]-[20]	[21]-[24]	[25]-[27]	[28]-[30]	[31]-[32]
Networked Microgrids	[33]-[36]	[37]-[40]	[41]-[42]	[43]-[44]	[45]
Microgrid Formation	[46]-[50]	[51]-[54]	[52]-[55]	[56]	[57]
Repair Scheduling	[58]	[59]		[58]-[61]	[62]-[65]

MILP: Mixed integer linear Program

Review: Repair and Restoration (1/3)

❖ MILP for **transmission** system repair and restoration (Arab 2015)

Assumptions

- Neglect travel time
- Crews are immediately present at the damaged components
- No specific crew assignments

Model

- Transmission system operation
- Repair schedule

Limited by number of crews

	Repair time			Repair status			
Line 1	1	1	1	0	0	0	0
Line 2	1	1	1	1	0	0	0
Line 3	0	0	0	1	1	0	0
Line 4	0	0	0	0	1	1	0

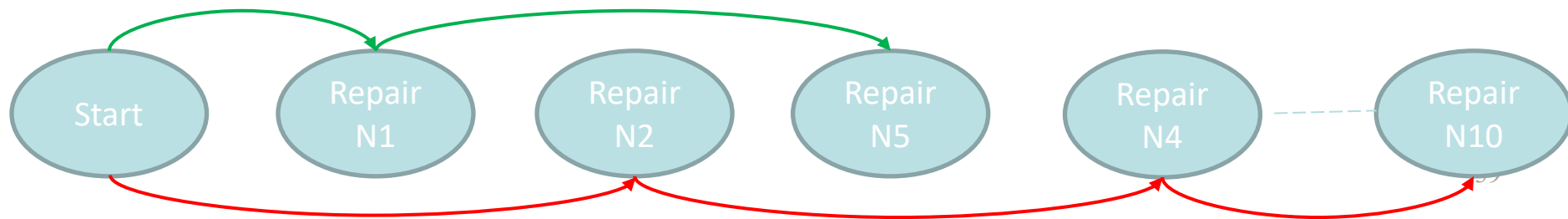
	Operation status						
Line 1	0	0	0	1	1	1	1
Line 2	0	0	0	0	1	1	1
Line 3	0	0	0	0	0	1	1
Line 4	0	0	0	0	0	0	1

Review: Repair and Restoration (2/3)

- ❖ A project by Los Alamos National Lab and National ICT Australia (NICTA), Australian National University.
- ❖ 2-Step approach for **transmission** systems (Pascal Van Hentenryck and Carlton Coffrin 2015):
 1. Restoration Ordering Problem: assume only one component can be repaired at each time step
 - ❖ Solved using MILP



2. Routing: solve a routing problem with precedence constraints
 - ❖ Solved using Constraint Programming
 - ❖ Precedence constraint



Review: Repair and Restoration (3/3)

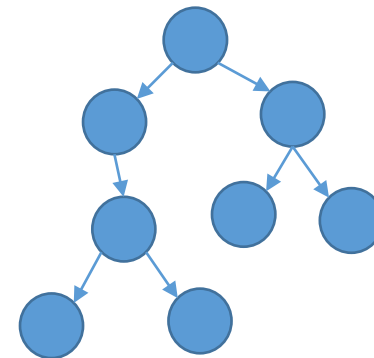
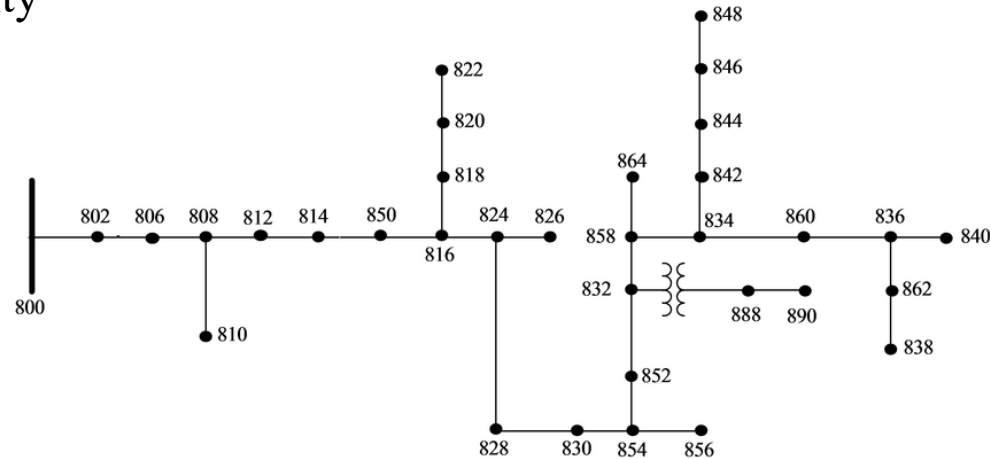
❖ Yushi Tan and Daniel S. Kirschen, University of Washington, 2017 (preprint).

• Assumptions

- Network is radial without switches.
- Power only from substation.
- Travel time is neglected.
- Power operation constraints are neglected.

• Method

- Solve scheduling problem (LP) to minimize the total weighted completion time under with “outtree” precedence constraint
 - obtain priority list
- Whenever a crew is free, select among the remaining candidate lines the one with the highest priority.



Research Objectives

What is missing?

- A co-optimization method that jointly optimizes crew routing and distribution system operation.
- Modeling fault isolation and tree/obstacle removal before repairing the lines.
- A preparation strategy before repair and restoration to ensure a fast response.

Objectives:

1. Proactive response: develop a stochastic program to pre-stage and prepare human resources and equipment before extreme weather events.
2. Develop MILP and stochastic mixed integer linear program (SMIP) models to co-optimize repair scheduling and the recovery operation of distribution systems.
3. Design solution algorithms for solving the above problems.

Outage scenario generation

- Weather forecast
- Fragility model

Pre-storm planning

- Choose staging locations
- Mobilize available crews and request assistance if necessary
- Obtain and allocate resources and equipment

Post-storm repair and restoration

- Coordinate tree and line crews
- Manage Equipment
- Isolate damaged components
- Operate generators and switches

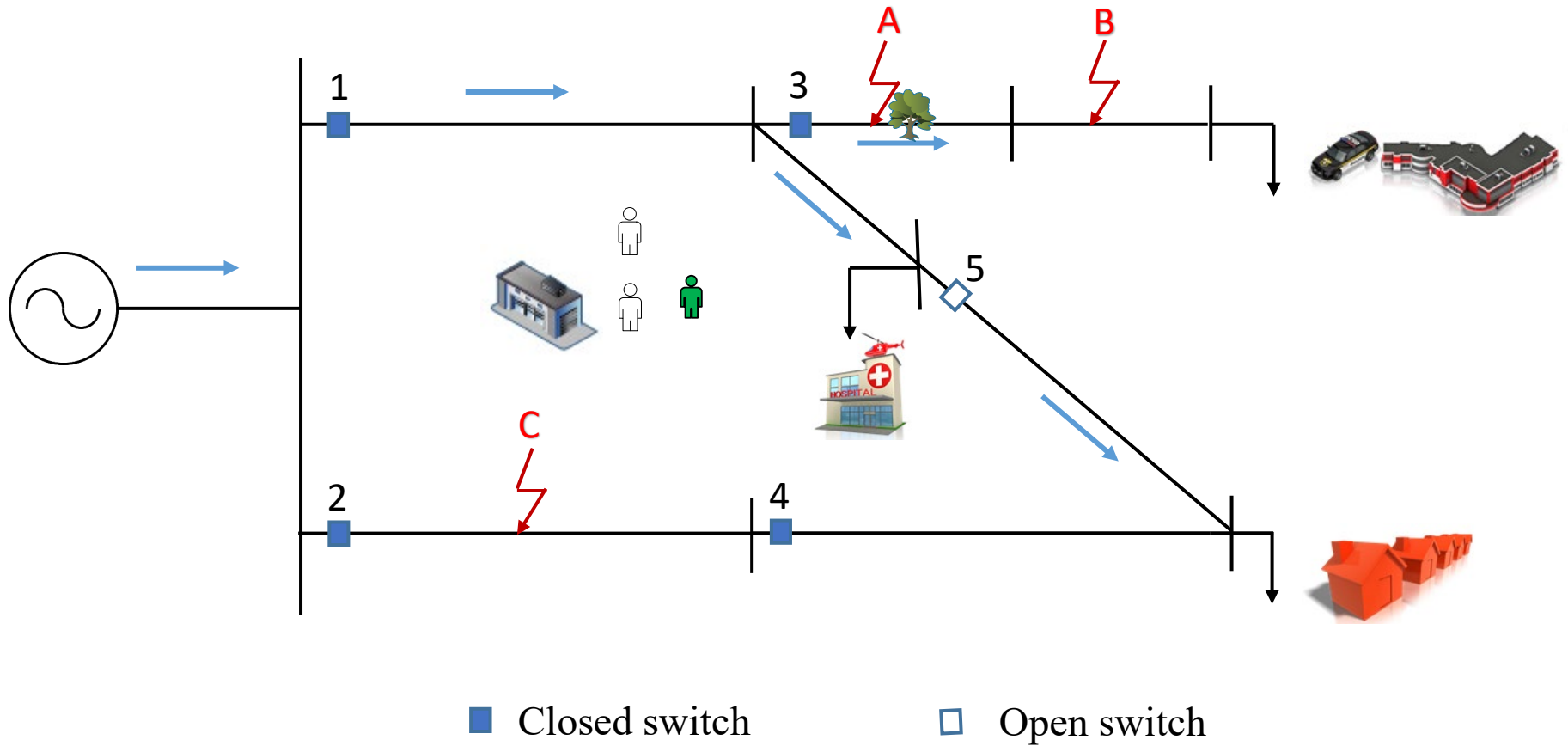
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Contributions

- A novel mathematical model for jointly optimizing the repair crew routing and distribution network operation problems is developed. The model can improve utilities' response to extreme events. Our research group is the first to develop a single mathematical model for co-optimizing crew routing and power restoration.
- A mathematical formulation is developed for fault isolation and service restoration. Isolation has been neglected in distribution system restoration studies that use mathematical programming.
- Development of efficient algorithms for solving the co-optimization problem.
 - Cluster-based decomposition
 - Priority-based decomposition
 - Hybrid mathematical programming and search algorithm
- 4 journal and 6 conference papers have been published, and 1 journal paper is under review.

Mathematical Modeling

Problem Overview



Mathematical Model

Distribution system repair and restoration problem (DSRRP)

Assumption:

Damage assessment has been conducted: the locations are known and the repair time is estimated.

Objective

- Minimize cost of shedding loads and switching operation

$$\min \sum_{\forall t} \left(\sum_{\forall \varphi} \sum_{\forall i} (1 - y_{i,t}) \rho_i^D P_{i,\varphi,t}^D + \rho^{SW} \sum_{k \in \Omega_{SW}} \gamma_{k,t} \right)$$

Constraints

- **Distribution system operations**
 - Power flow
 - Cold-load pickup
 - Voltage constraints
 - Reconfiguration and fault isolation constraints
- **Crew routing**
 - Path-flow constraints
 - Start/end location
 - A damaged line is repaired by one crew
 - Arrival (repair start) time
 - Tree removal before line repair
 - Equipment constraints

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Distribution System

1. Generator limits
2. Line limits
3. Node balance
4. Kirchhoff voltage law
(Chen 2018)
 - Losses are neglected
5. Voltage regulators

$$1 \quad \begin{cases} 0 \leq P_{i,\varphi,t}^G \leq P_i^{Gmax}, \forall i, \varphi, t \\ 0 \leq Q_{i,\varphi,t}^G \leq Q_i^{Gmax}, \forall i, \varphi, t \end{cases}$$

$$2 \quad \begin{cases} -u_{k,t} p_{k,\varphi} P_k^{Kmax} \leq P_{k,\varphi,t}^K \leq u_{k,t} p_{k,\varphi} P_k^{Kmax}, \forall k, \varphi, t \\ -u_{k,t} p_{k,\varphi} Q_k^{Kmax} \leq Q_{k,\varphi,t}^K \leq u_{k,t} p_{k,\varphi} Q_k^{Kmax}, \forall k, \varphi, t \end{cases}$$

$$3 \quad \begin{cases} \sum_{\forall k \in K(.,i)} P_{k,\varphi,t}^K + P_{i,\varphi,t}^G = \sum_{\forall k \in K(i,.)} P_{k,\varphi,t}^K + P_{i,\varphi,t}^L, \forall i, \varphi, t \\ \sum_{\forall k \in K(.,i)} Q_{k,\varphi,t}^K + Q_{i,\varphi,t}^G = \sum_{\forall k \in K(i,.)} Q_{k,\varphi,t}^K + Q_{i,\varphi,t}^L, \forall i, \varphi, t \end{cases}$$

$$4 \quad \begin{cases} U_{j,t} - U_{i,t} + \bar{Z}_k S_k^* + \bar{Z}_k^* S_k \leq (2 - u_{k,t} - p_k)M, \forall k \in \Omega_L \setminus \Omega_V, t \\ U_{j,t} - U_{i,t} + \bar{Z}_k S_k^* + \bar{Z}_k^* S_k \geq -(2 - u_{k,t} - p_k)M, \forall k \in \Omega_L \setminus \Omega_V, t \end{cases}$$

$$5 \quad \begin{cases} -(2 - u_{k,t} - p_k)M \leq a_k^2 U_{j,t} - U_{i,t}, \forall k \in \Omega_V, t \\ a_k^2 U_{j,t} - U_{i,t} \leq (2 - u_{k,t} - p_k)M, \forall k \in \Omega_V, t \end{cases}$$

u : status of the line
 p_k : for line k with phases a, c ,
 $p_k = [1,0,1]$
 $U = V^2$
 a_k : the ratio between the
 primary and secondary winding
 for each phase

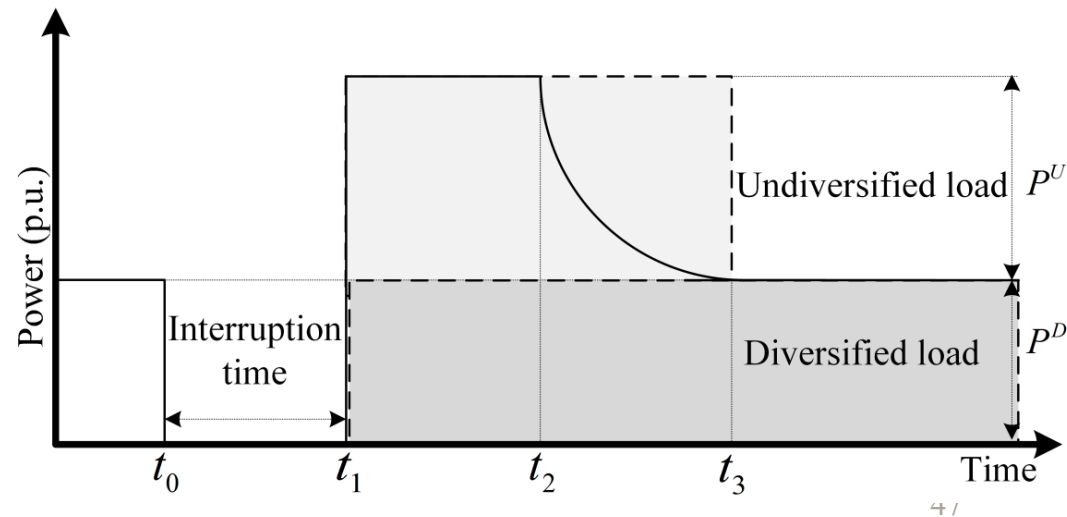
Cold-Load Pickup

- Cold load pickup (CLPU) is the well-known problem defined as excessive inrush current drawn by loads when the distribution circuits are re-energized after extended outages.
- The typical behaviour of CLPU can be represented using a **delayed exponentially decaying function**.
- We use two blocks to provide a conservative approach and guarantee the supply-load balance (Liu, PSERC 2009).

$$P_{i,\varphi,t}^L = y_{i,t}P_{i,\varphi,t}^D + (y_{i,t} - y_{i,\max(t-\lambda,0)})P_{i,\varphi,t}^U, \quad \forall i, \varphi, t$$

$$Q_{i,\varphi,t}^L = y_{i,t}Q_{i,\varphi,t}^D + (y_{i,t} - y_{i,\max(t-\lambda,0)})Q_{i,\varphi,t}^U, \quad \forall i, \varphi, t$$

- λ : number of time steps required for the load to return to normal condition.
- If a load goes from a de-energized state $y = 0$, to an energized state $y = 1$, it will go back to normal condition after λ .



Voltage Regulator

- Voltage regulator with variable tap setting
- Voltage on the secondary side = $a \times$ voltage on the primary
- The standard voltage regulator provides $\pm 10\%$ adjustment in thirty-two 0.625% steps
- $a = [1 + 0.00625 \times Tap] \rightarrow U_j = [1 + 0.00625 \times Tap]^2 U_i$
- Tap = -16, -15, ..., 16
- Define variable $\tau \in \{0,1\}^{33}$, where $\tau_1 = 1 \rightarrow Tap = -16$
- $r = a^2 = [0.8100, 0.8213, \dots, 1.2100]$
- Exact linear constraints
$$-M(1 - \tau_p) + r_p U_i \leq U_j \leq r_p U_i + M(1 - \tau_p), \forall \text{ voltage regulators, } p \in \{1..33\}$$
- Example: if a^2 is desired to be 0.81, then $\tau_p = 1$, if $p = 1$
$$0.81 U_i \leq U_j \leq 0.81 U_i$$
- Simplified constraint
$$0.81 U_i \leq U_j \leq 1.21 U_i$$

Reconfiguration

1. Radiality constraints (for radial networks)

$$\sum_{k \in \Omega_{K(l)}} u_{k,t} \leq |\Omega_{K(l)}| - 1, \forall l, t$$

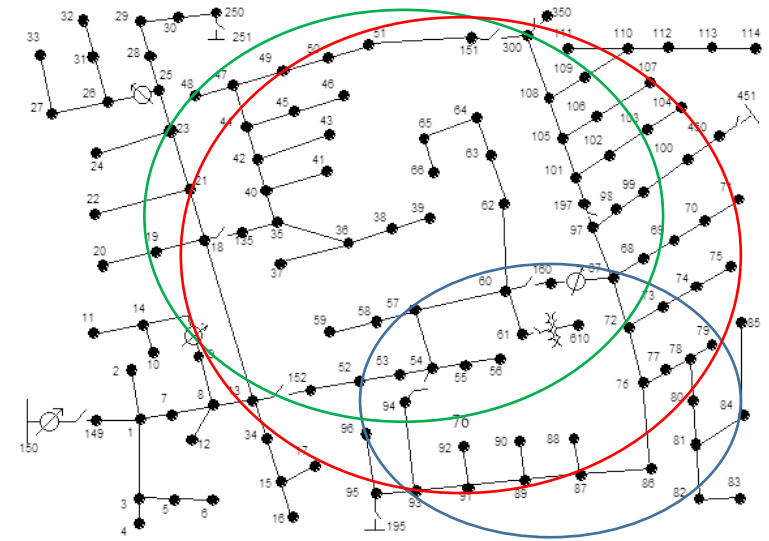
2. Count switching operations

$$\gamma_{k,t} \geq u_{k,t} - u_{k,t-1}, \forall k \in \Omega_{SW}, t$$

$$\gamma_{k,t} \geq u_{k,t-1} - u_{k,t}, \forall k \in \Omega_{SW}, t$$

3. Fault Isolation:

- Force the voltage to be zero on damaged lines
- The voltage propagates through KVL until a CB/switch stops the propagation



(Borghetti 2012)

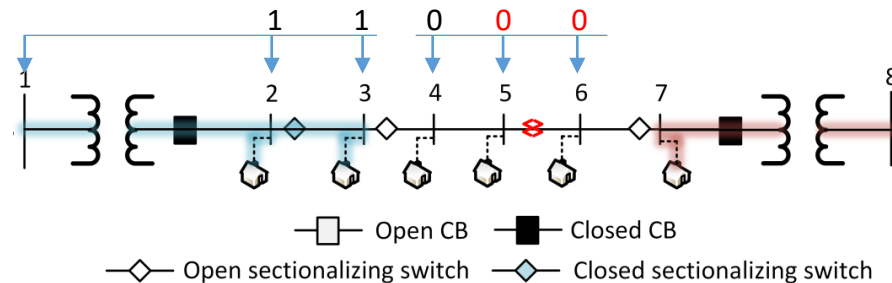
χ : outage status of bus
 $\Omega_{K(l)}$: set of lines in loop l
 Ω_{DK} : set of damaged lines
 γ : Binary parameter equals one if a switch changes its status

$$2u_{k,t} \geq \chi_{i,t} + \chi_{j,t}, \forall k \in \Omega_{DK}, t$$

$$\chi_{i,t} U_{min} \leq U_{i,t} \leq \chi_{i,t} U_{max}, \forall i, t$$

$$\chi_{i,t} \geq y_{i,t}, \forall i, t$$

$$u_{k,t} = 1, \forall k \notin \{\Omega_{SW} \cup \Omega_{DK}\}, t$$



Crew Routing (1/2)

Vehicle routing problem (VRP)

1. Starting and ending locations
2. Path-flow constraint
3. A damaged component is visited only once by a line crew and a tree crew (if required)

x : binary var equals 1 if crew travels the path
 $\phi^{0/1}$: start/return location time
 N : set of nodes
 Ω_{DT} : set of lines damaged by trees
 $C^{L/T}$: set of line/tree crews

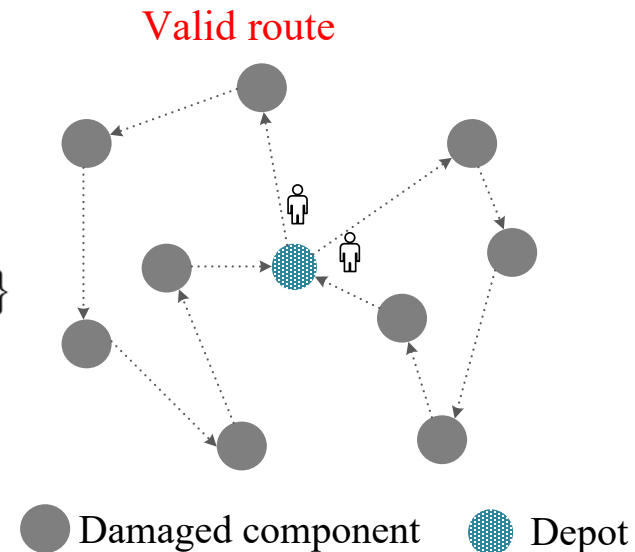
$$\sum_{\forall m \in N} x_{\phi_c^0, m, c} = 1, \forall c$$

$$\sum_{\forall m \in N} x_{m, \phi_c^1, c} = 1, \forall c$$

$$\sum_{\forall n \in N \setminus \{m\}} x_{m, n, c} - \sum_{\forall n \in N \setminus \{m\}} x_{n, m, c} = 0, \forall c, m \in N \setminus \{\phi_c^0, \phi_c^1\}$$

$$\sum_{\forall c \in C^L} \sum_{\forall m \in N \setminus \{n\}} x_{m, n, c} = 1, \forall n \in \Omega_{DK}$$

$$\sum_{\forall c \in C^T} \sum_{\forall m \in N \setminus \{n\}} x_{m, n, c} = 1, \forall n \in \Omega_{DT}$$



Crew Routing (2/2)

1. Calculate arrival time
 $Arrival_n = Arrival_m + Travel_{mn} + Repair_m$
2. Tree crews must finish before the line crews start repairing
3. Set arrival time = 0 (empty) if a crew does not visit a component
4. Crews must have enough equipment to repair the components
5. Each crew has a capacity
6. Equipment are used/picked up as the crews travel between components
 Equipment on hand = equipment at previous location – equipment used
7. The equipment is taken from the depot/warehouse

$$\begin{aligned}
 1 \quad & \left[\begin{aligned} & \alpha_{m,c} + T_{m,c} + tr_{m,n} - (1 - x_{m,n,c}) M \leq \alpha_{n,c} \\ & \forall m \in N \setminus \{\phi_c^1\}, n \in N \setminus \{\phi_c^0, m\}, c \end{aligned} \right. \\
 2 \quad & \left[\begin{aligned} & \sum_{c \in C^L} \alpha_{m,c} \geq \sum_{c \in C^T} \alpha_{m,c} + T_{m,c} \sum_{\forall n \in N} x_{m,n,c}, \forall m \in \Omega_{DT} \end{aligned} \right. \\
 3 \quad & \left[\begin{aligned} & 0 \leq \alpha_{m,c} \leq M \sum_{n \in N} x_{n,m,c}, \forall m \in N \setminus \{\phi_c^0, \phi_c^1\}, c \end{aligned} \right. \\
 4 \quad & \left[\begin{aligned} & \sum_{\forall n \in N} x_{n,m,c} \mathcal{R}_{m,r} \leq E_{c,m,r}, \forall m, r, c \in C^L \end{aligned} \right. \\
 5 \quad & \left[\begin{aligned} & \sum_{\forall r} Cap_r^R E_{c,m,r} \leq Cap_c^C, \forall m, c \in C^L \end{aligned} \right.
 \end{aligned}$$

$$\begin{aligned}
 6 \quad & \left[\begin{aligned} & -M(1 - x_{m,n,c}) \leq E_{c,m,r} - \mathcal{R}_{m,r} - E_{c,n,r} \leq M(1 - x_{m,n,c}), \\ & \forall m \in N \setminus \{\phi_c^1\}, n \in N \setminus \{\phi_c^0, m\}, c \in C^L, r \\ & -M(1 - x_{w,n,c}) \leq E_{c,w,r} + Res_{c,w,r}^C - E_{c,n,r} \leq M(1 - x_{w,n,c}), \\ & \forall w, n \in N \setminus \{\phi_c^0, \phi_c^1, w\}, c \in C^L, r \\ & -M(1 - x_{\phi_c^0, n, c}) \leq Res_{c, \phi_c^0, r}^C - E_{c,n,r} \leq M(1 - x_{\phi_c^0, n, c}), \forall n \in N \setminus \{\phi_c^0\}, c \in C^L, r \end{aligned} \right. \\
 7 \quad & \left[\begin{aligned} & Res_{w,r}^D \geq \sum_{\forall c \in C^L, \phi_c^0 = w} Res_{c, \phi_c^0, r}^C + \sum_{\forall c \in C^L} Res_{c,w,r}^C, \forall w, r \end{aligned} \right.
 \end{aligned}$$

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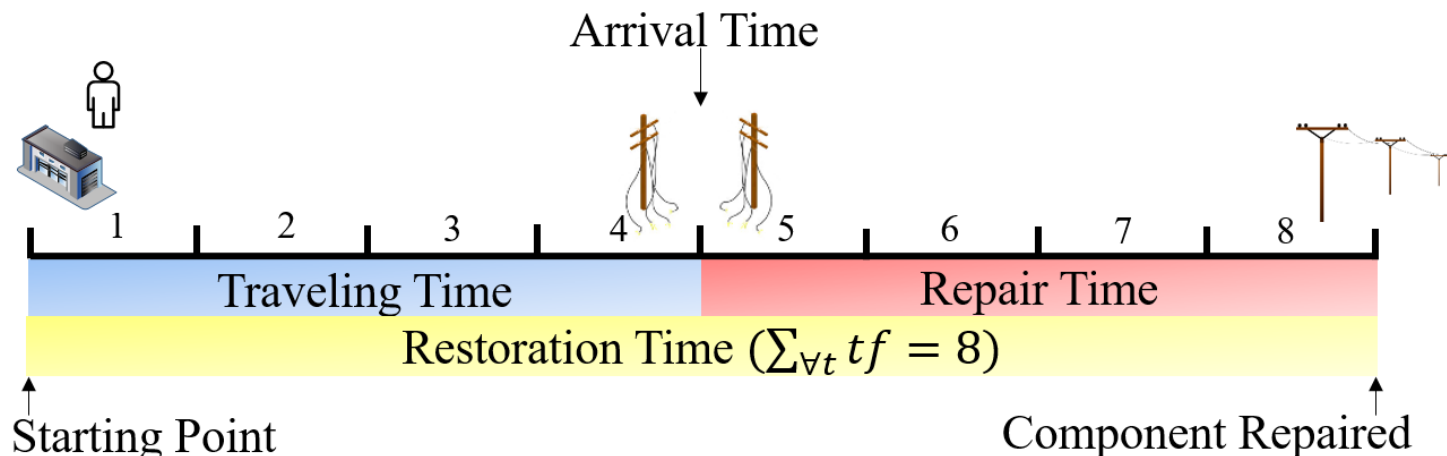
Connecting Operation and Routing

- When can we operate the component?
 - Define binary variable f which equals 1 once the line is repaired
 - Calculate the restoration time (Arrival time + Repair time)
 - Set the status of the line ($u_{k,t}$) to 1 once the line is repaired

$$\sum_{\forall t} f_{m,t} = 1, \forall m \in \Omega_D$$

$$\sum_{\forall t} t f_{m,t} \geq \sum_{\forall c} (\alpha_{m,c} + \mathcal{T}_{m,c} \sum_{\forall n \in N} x_{m,n,c}), \forall m \in \Omega_D$$

$$u_{m,t} = \sum_{\tau=1}^t f_{m,\tau}, \forall m \in \Omega_{DL}, t$$



Challenges

- VRP is NP-hard, obtaining the optimal solution for large cases is very challenging.
- VRP is commonly solved using heuristic methods.
- Combining VRP with distribution system operation highly increases the complexity.
- Large number of damages:
 - Routing becomes extremely difficult

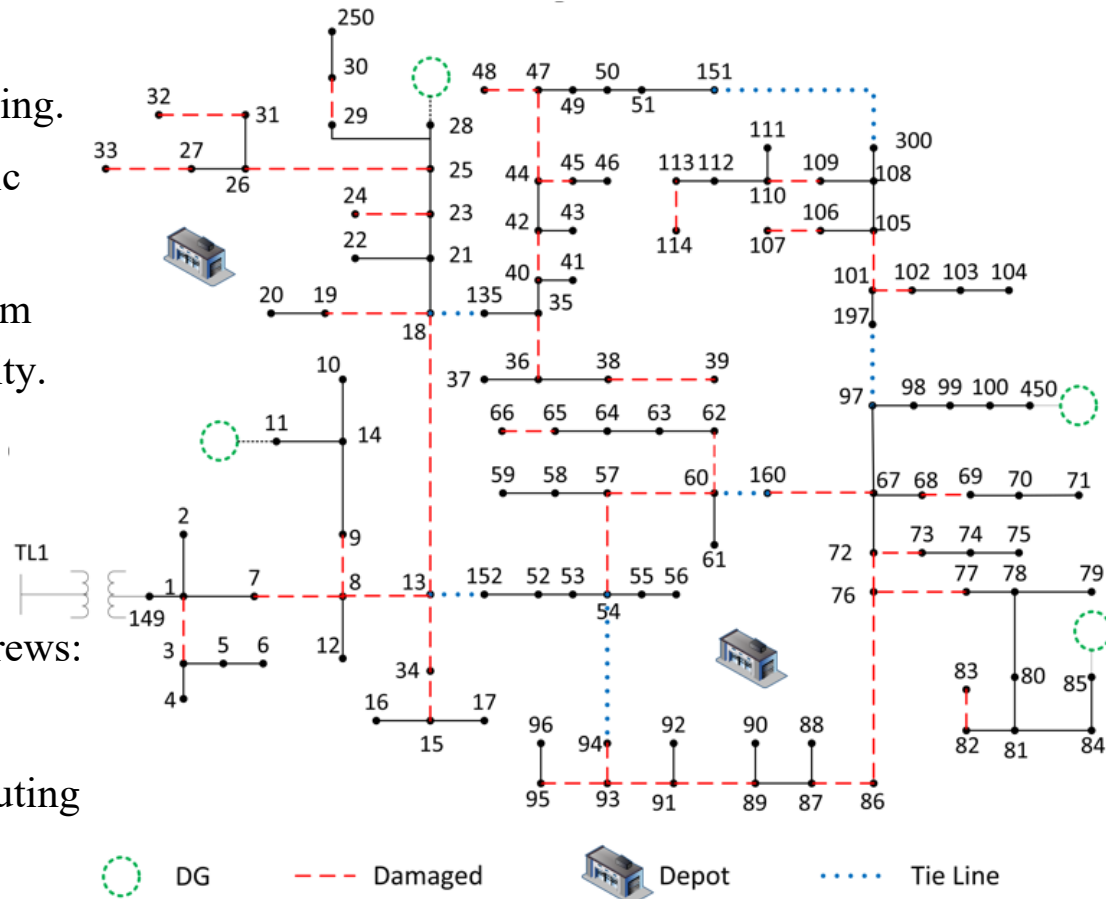
E.g. 30 damaged components and 10 crews:

$$x_{m,n,c} \rightarrow 30 \times 30 \times 10 =$$

→ 9000 integer variables for routing

only

- **Computation time is critical!**



Proposed Solution Algorithms

- Direct method
 - Use commercial solvers (e.g., CPLEX, GUROBI) to solve the mathematical model
- Priority-based
- Cluster-based (C-DSRRP)
- Assignment-based (A-DSRRP)
- A-DSRRP → Neighborhood Search

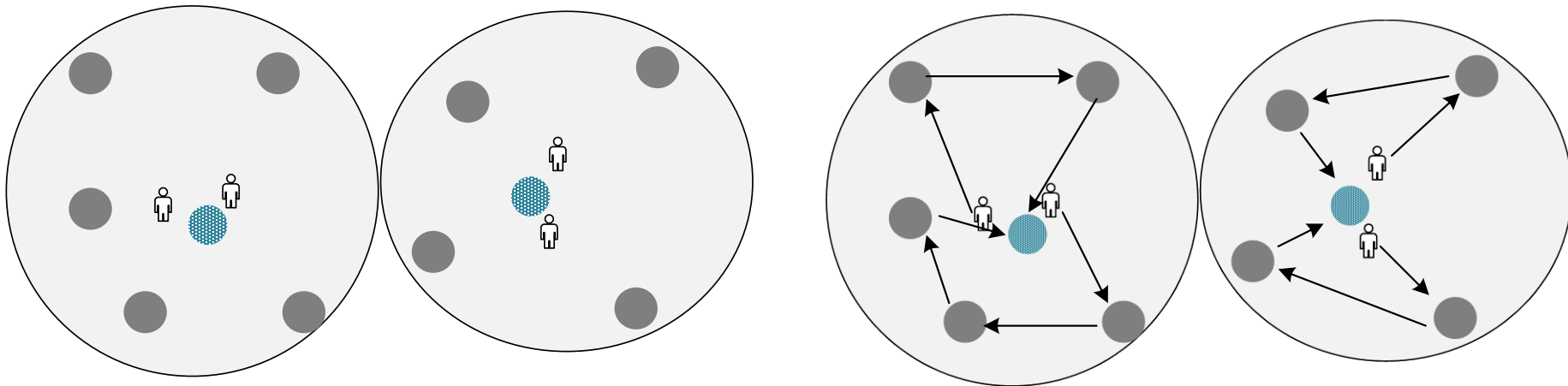
Priority-based

- The goal of this method is to mimic the approach used in practice
- Define the priority of the lines
 1. Repair lines connected to high-priority customers.
Weight factor $W_1 = 10$
 2. Repair 3-phase lines.
Weight factor $W_2 = 5$
 3. Repair single phase lines and individual customers.
Weight factor $W_3 = 1$
- Identify the lines that must be repaired to restore high-priority customers.
 - $\min\{(\text{number of lines to repair}) \mid \text{s.t. operation constraints}\}$
- Solve the crew routing problem
 - $\min\{(\sum_{\forall p} \sum_{k \in L_p} \sum_{c \in C^L} W_p \alpha_{c,k}) \mid \text{s.t. routing constraints}\}$

L_p : set of lines to repair with priority p

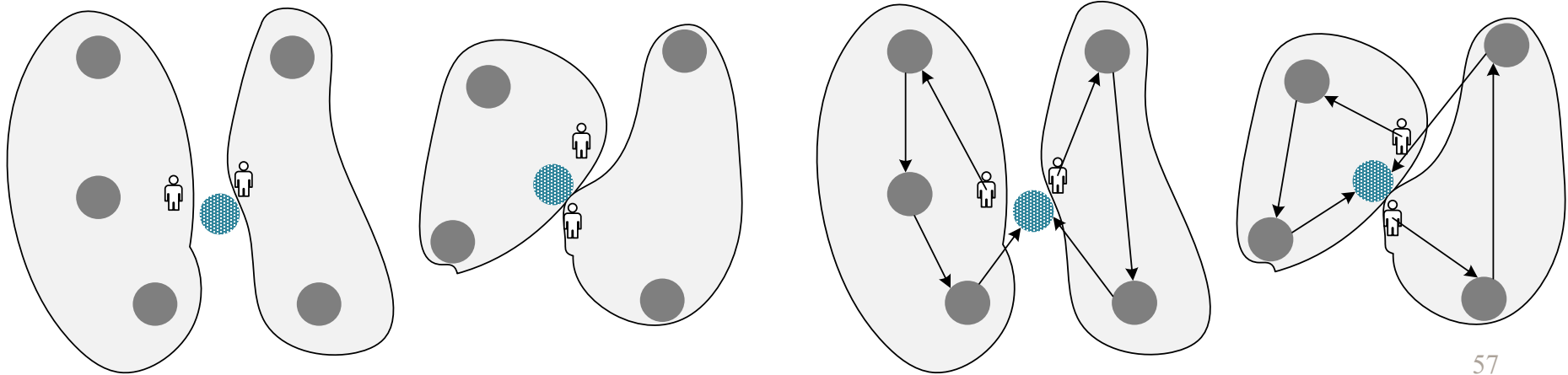
Cluster-based

- Cluster the damaged components to depots.
 - $\min \{(\text{distance between depots and components}) \mid \text{s.t. resource constraint}\}$
 - C-DSRRP
 - Solve DSRRP with the crews routed based on the clusters.
- VRP problem \rightarrow Multi-VRP subproblems



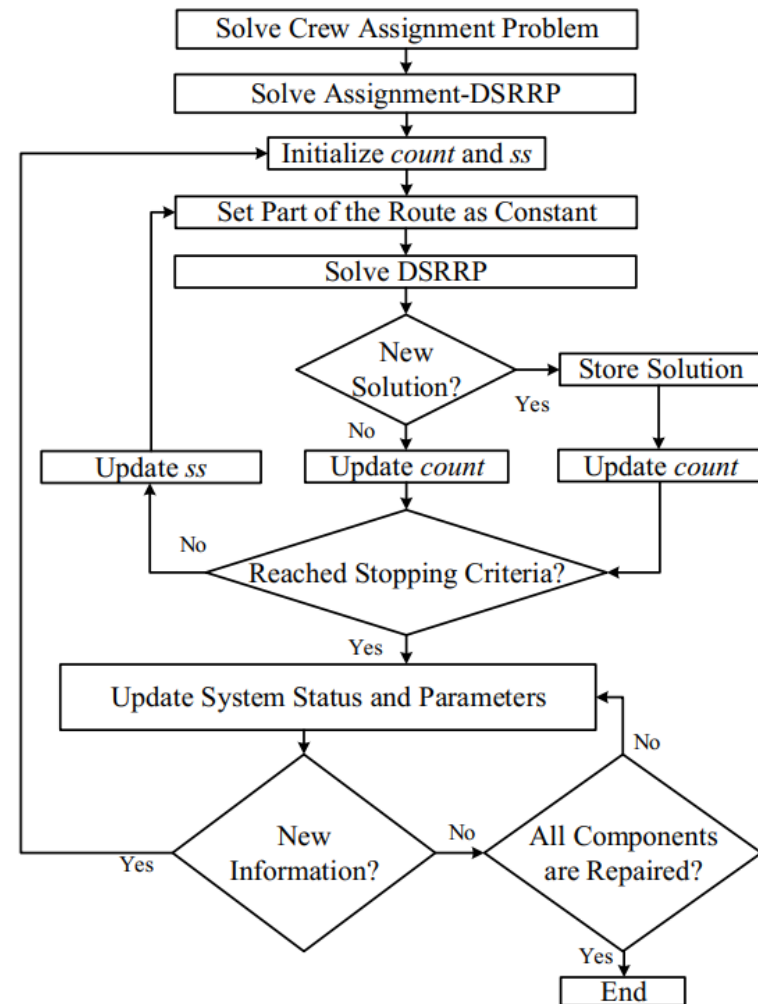
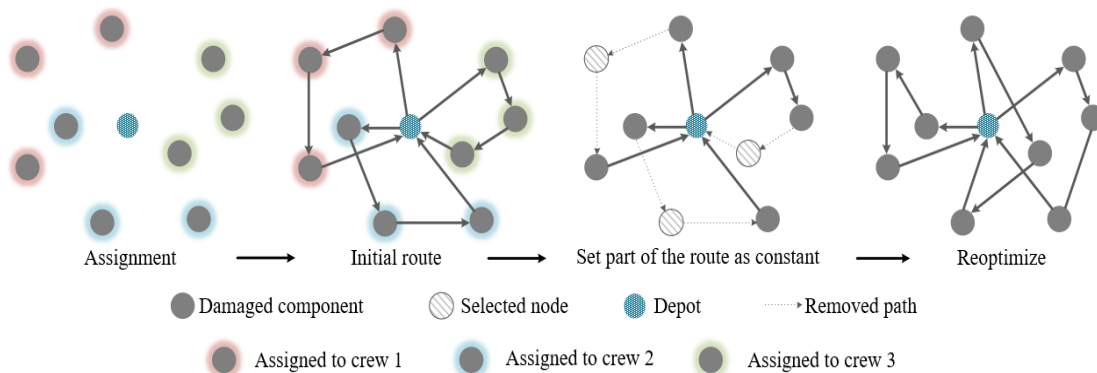
Assignment-based

- Assign the damaged components to crews.
 - $\min \{(\text{distances between components that are assigned to the crews}) \mid \text{s.t. resource constraint and assignment constraints}\}$
- A-DSRRP
 - Solve DSRRP with the crews routed based on the assignments.
- VRP problem \rightarrow Multi-TSP subproblems



Reoptimization (A-DSRRP → Large Neighborhood Search)

1. Select *ss* nodes (damaged components)
2. Remove part of the route connected to the selected components
3. Set rest of the route to be constant
4. Solve the optimization problem DSRRP (with *warm start* and limit 120 s)
5. Repeat until we reach the stopping criteria (increase *ss* after *count* iterations without change)
6. Update the route once new information is obtained

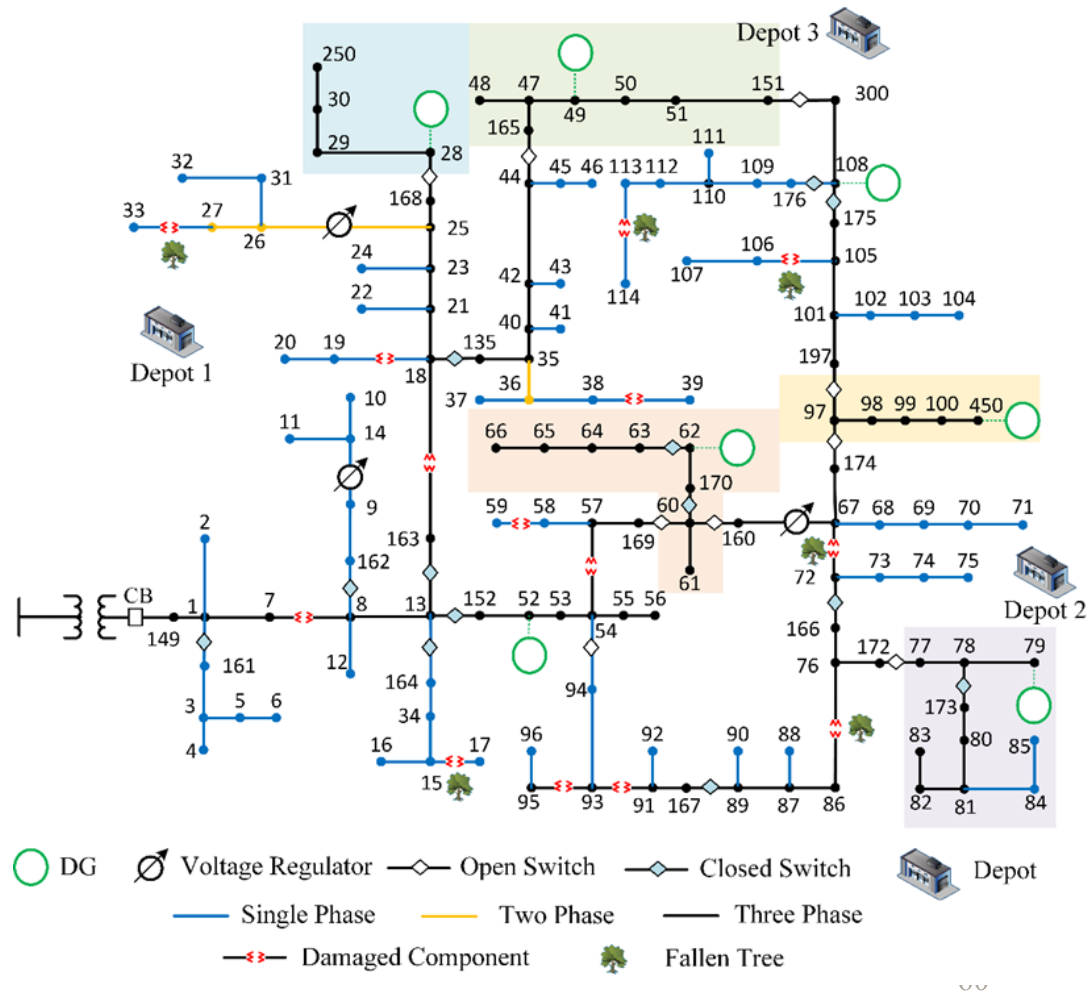


Algorithm

- Use assignment-based approach
- Subproblem I:
 - Assign the damaged components to the crews
 - Consider uncertainty of the repair times
 - Solve using the extensive-form
- Subproblem II
 - Solve stochastic DSRRP with the crews dispatched to the assigned damaged components
 - Use Progressive Hedging to solve the stochastic DSRRP model

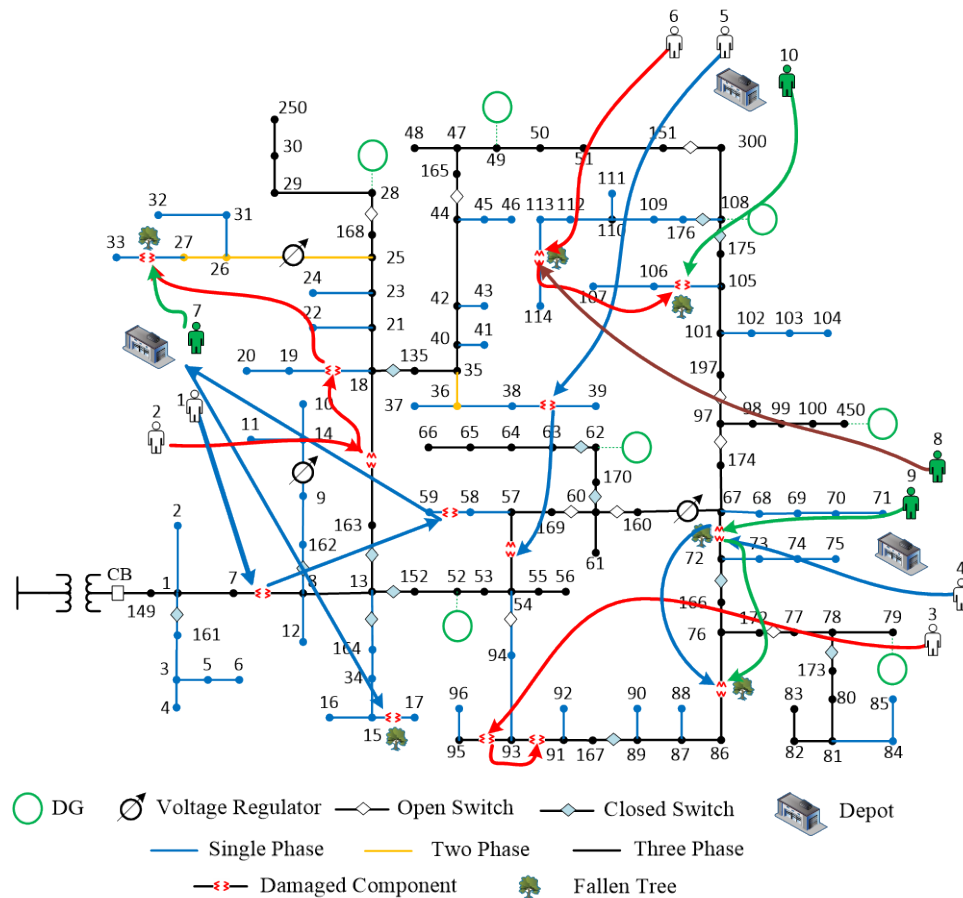
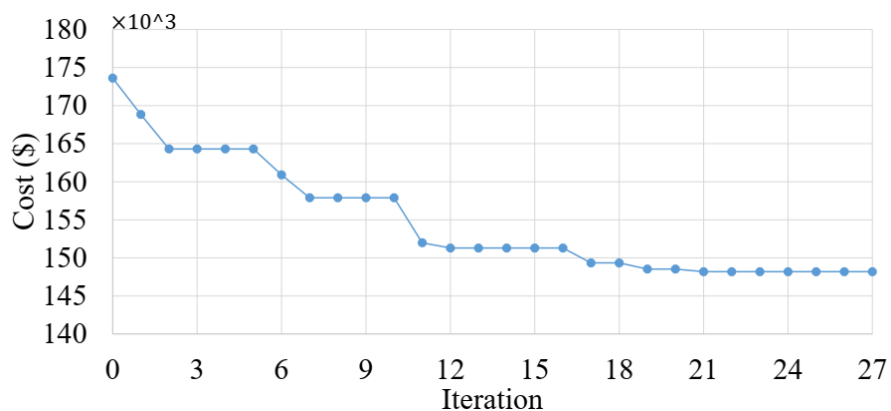
Test Case

- Modified IEEE 123-bus distribution feeder.
- 7 dispatchable DGs and 18 new switches are installed.
- Loads at buses 30, 48, 49, 53, 65, and 76 are critical loads.
- 3 depots, 6 line crews, and 4 tree crews.
- 14 damaged lines.
- Repair times
 - Intensity of the damage is represented by the repair time.
 - Repair time is generated using a truncated lognormal distribution (Z. Zhu 2012).
- Time limit 3600 seconds (Van Hentenryck 2011).
- Solved using AMPL-CPLEX.

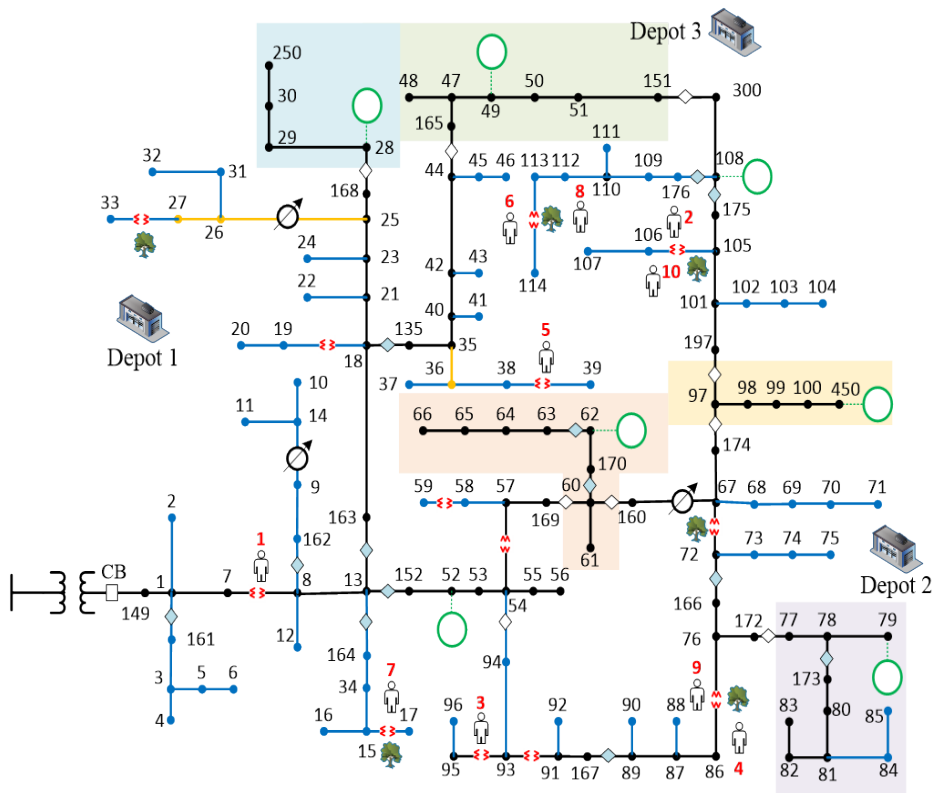


Simulation Results-Reoptimization

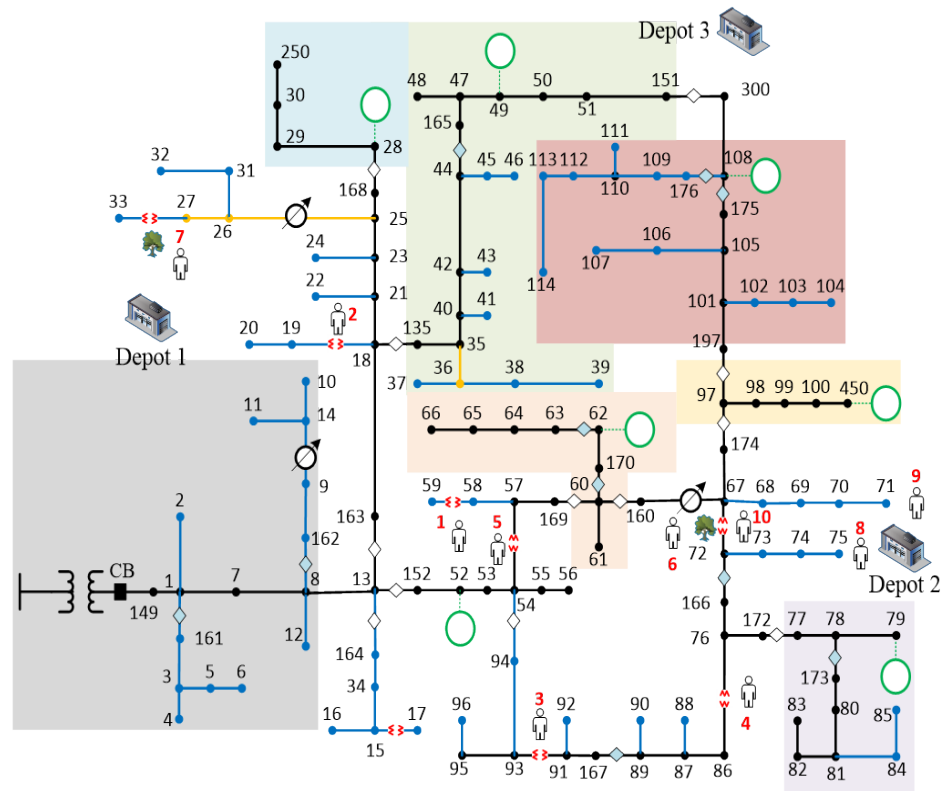
- Objective: \$148,185
- Energy served = 62,436 kWh
- All loads are served after 9 hours
- Iterations: 27
- Computation time: 3120 seconds



Simulation Results-Reoptimization (Cont.)

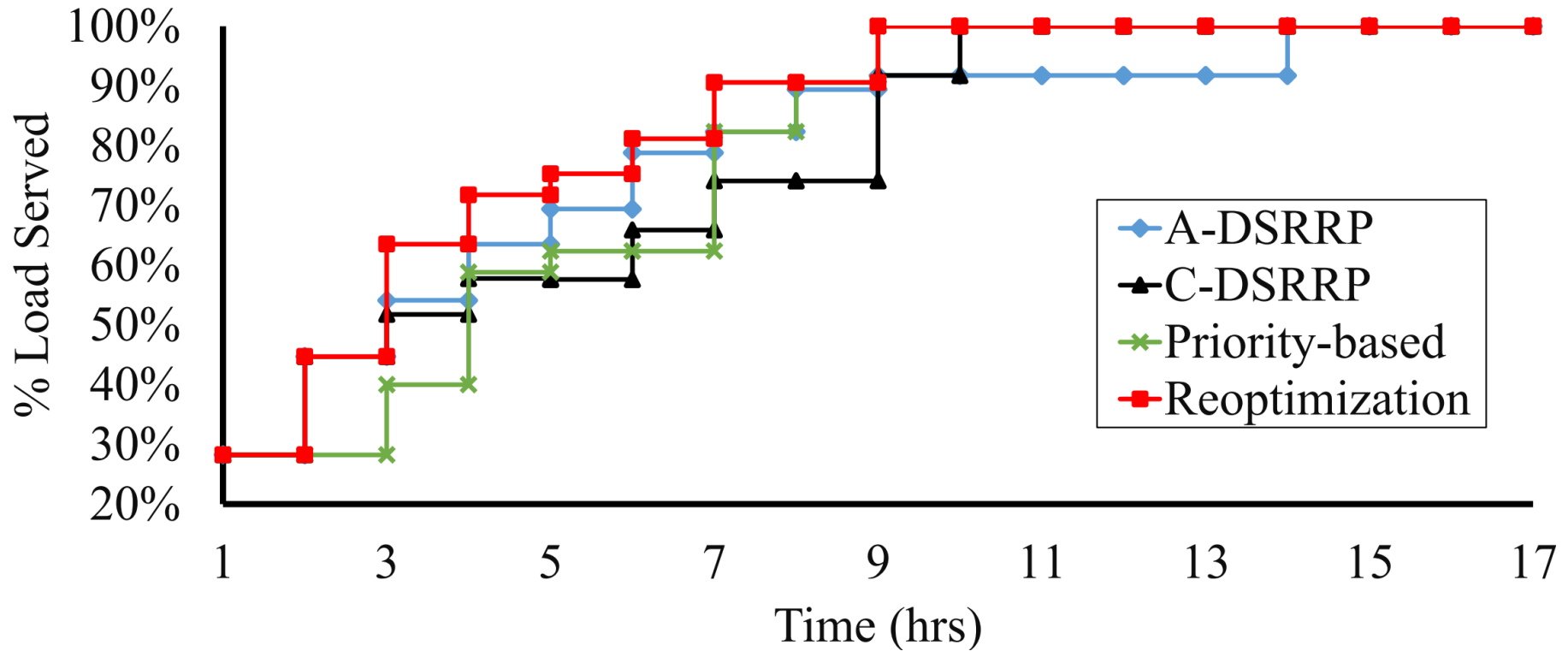


After 2 hours



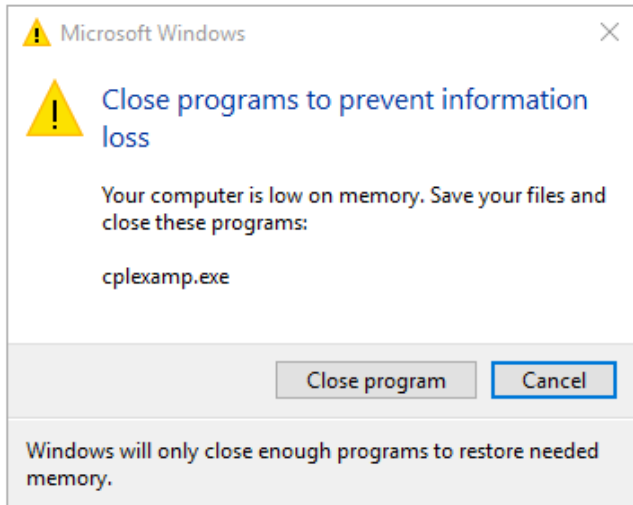
After 4 hours

Solution Comparison



Optimality Gap

- Using the **Reoptimization solution as an initial solution** (warm start), the complete DSRRP problem is solved using CPLEX.
- Out of memory after approximately 4 hours.
- Solution did not change, optimality gap is 4.28%



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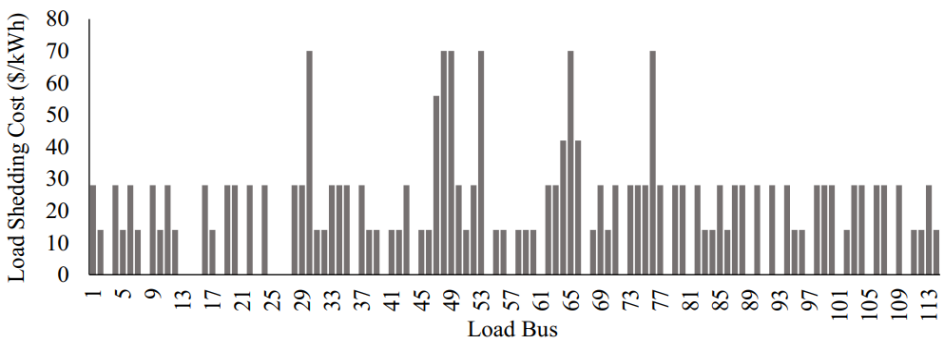
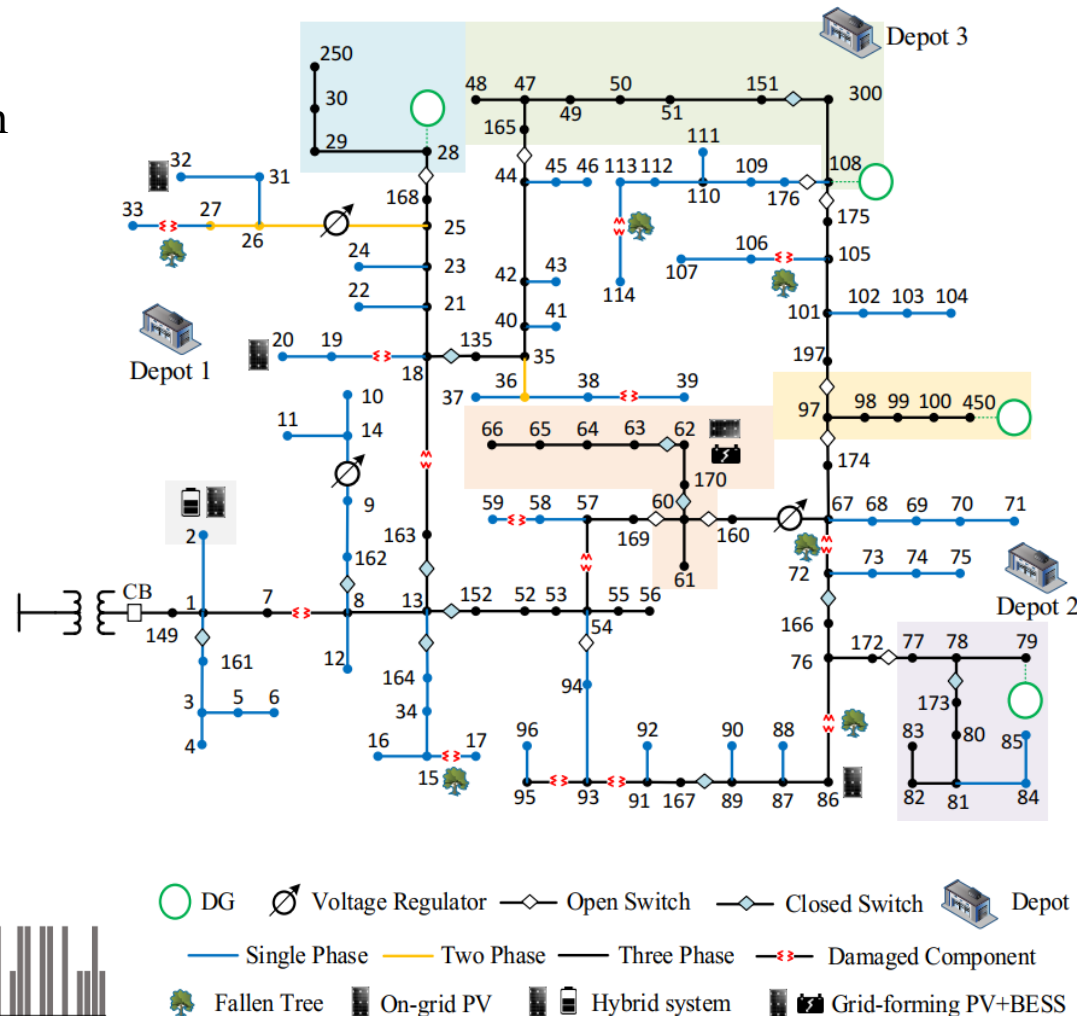
Nodefile size = 66619.10 MB <46874.73 MB after compression>
244130 153069   infeasible           148185.1360    141836.3600    22545539    4.28%
244607 153442   infeasible           148185.1360    141836.3600    22580874    4.28%
244963 153719   146770.1280        39    148185.1360    141836.3600    22612269    4.28%
245344 154013   144319.8784        52    148185.1360    141836.3600    226242396   4.28%
245688 154250   146310.8160        50    148185.1360    141836.3600    22687694    4.28%
246529 154880   144698.5096       150    148185.1360    141836.3600    22765300    4.28%
247242 155524   cutoff              148185.1360    141836.3600    22802302    4.28%
247788 155985   141836.3600        45    148185.1360    141836.3600    22838488    4.28%
248329 156415   145778.3120        46    148185.1360    141836.3600    22881632    4.28%
248420 156456   145827.7040        42    148185.1360    141836.3600    22896153    4.28%
Elapsed time = 11676.98 sec. <2903518.04 ticks, tree = 70590.00 MB>
Nodefile size = 68446.91 MB <48171.65 MB after compression>
249169 157107   141836.3600        45    148185.1360    141836.3600    22940872    4.28%
250130 158005   144808.1120       105    148185.1360    141836.3600    22975578    4.28%
250283 158131   148106.5841        52    148185.1360    141836.3600    22989440    4.28%
251257 159007   infeasible           148185.1360    141836.3600    23051126    4.28%
251705 159381   143775.0800        41    148185.1360    141836.3600    23086397    4.28%
252218 159781   147645.9195        43    148185.1360    141836.3600    23135082    4.28%
252432 159915   147781.9443        65    148185.1360    141836.3600    23166657    4.28%
252849 160274   142378.5863        48    148185.1360    141836.3600    23194499    4.28%
253748 161107   141836.3600        75    148185.1360    141836.3600    23239965    4.28%
254357 161627   cutoff              148185.1360    141836.3600    23274856    4.28%
Elapsed time = 12041.41 sec. <2943675.72 ticks, tree = 72743.32 MB>
Nodefile size = 70567.62 MB <49657.44 MB after compression>
254817 161992   144786.0480        44    148185.1360    141836.3600    23307683    4.28%
255650 162669   144792.8800        28    148185.1360    141836.3600    23358873    4.28%
256012 162903   146721.6712        56    148185.1360    141836.3600    23405846    4.28%
256627 163438   142808.8000        39    148185.1360    141836.3600    23440388    4.28%
256745 163527   142162.6720        86    148185.1360    141836.3600    23451502    4.28%
257371 164020   141897.6968        84    148185.1360    141836.3600    23504809    4.28%
258059 164622   142808.8000        65    148185.1360    141836.3600    23554037    4.28%
258396 164881   144005.7854        65    148185.1360    141836.3600    23598143    4.28%
259217 165522   144089.8707        55    148185.1360    141836.3600    23671598    4.28%
259801 165904   141893.1540        81    148185.1360    141836.3600    23734151    4.28%
Elapsed time = 12273.94 sec. <2988915.68 ticks, tree = 74955.27 MB>
Nodefile size = 72812.13 MB <51212.47 MB after compression>
259913 165940   147669.0816        60    148185.1360    141836.3600    23750898    4.28%
260150 166121   142088.9173        98    148185.1360    141836.3600    23775439    4.28%
260911 166724   141893.1540        69    148185.1360    141836.3600    23834279    4.28%
261455 167185   142162.6720        38    148185.1360    141836.3600    23873510    4.28%
262071 167669   146307.5818        89    148185.1360    141836.3600    23925056    4.28%
262911 168410   142485.6800        23    148185.1360    141836.3600    23971510    4.28%
263452 168856   141836.3600        43    148185.1360    141836.3600    24014071    4.28%
263988 169269   148087.6400        64    148185.1360    141836.3600    24062484    4.28%
264770 169872   cutoff              148185.1360    141836.3600    24126951    4.28%
265063 170041   cutoff              148185.1360    141836.3600    24153287    4.28%
Elapsed time = 12694.31 sec. <3029516.40 ticks, tree = 76841.88 MB>
Nodefile size = 74700.20 MB <52523.82 MB after compression>
265279 170216   145802.6160        79    148185.1360    141836.3600    24174500    4.28%
265909 170629   143527.5600        38    148185.1360    141836.3600    24237512    4.28%

```

There may be further error information in the clone logs.

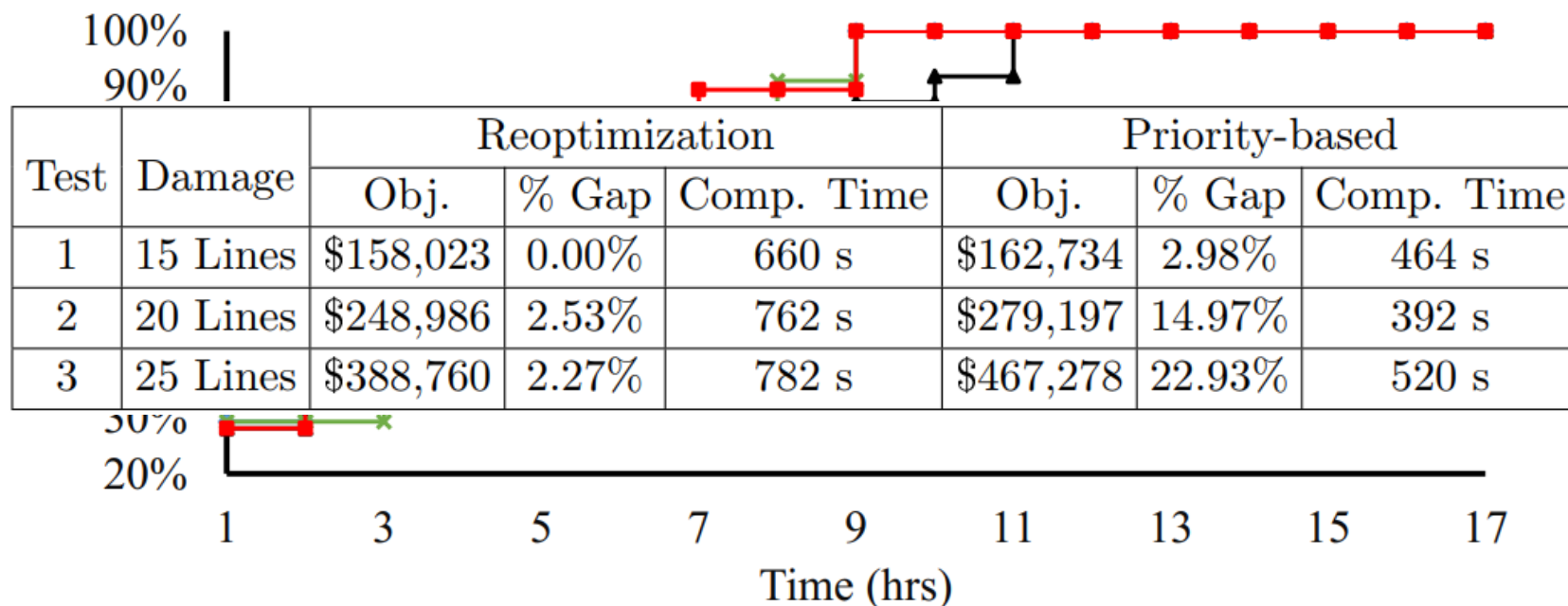
Test Case

- Modified IEEE 123-bus distribution feeder.
- 9 DGs and 23 switches
- 3 depots, 6 line crews, and 4 tree crews.
- 14 damaged lines
- The model and algorithm are implemented in AMPL, with CPLEX solver



Results: Solution Comparison

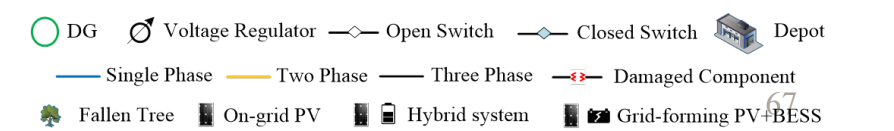
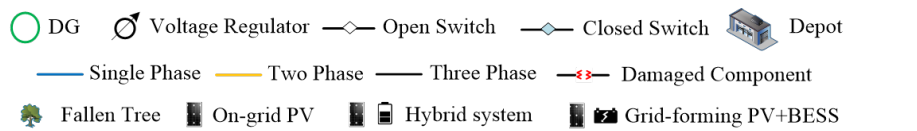
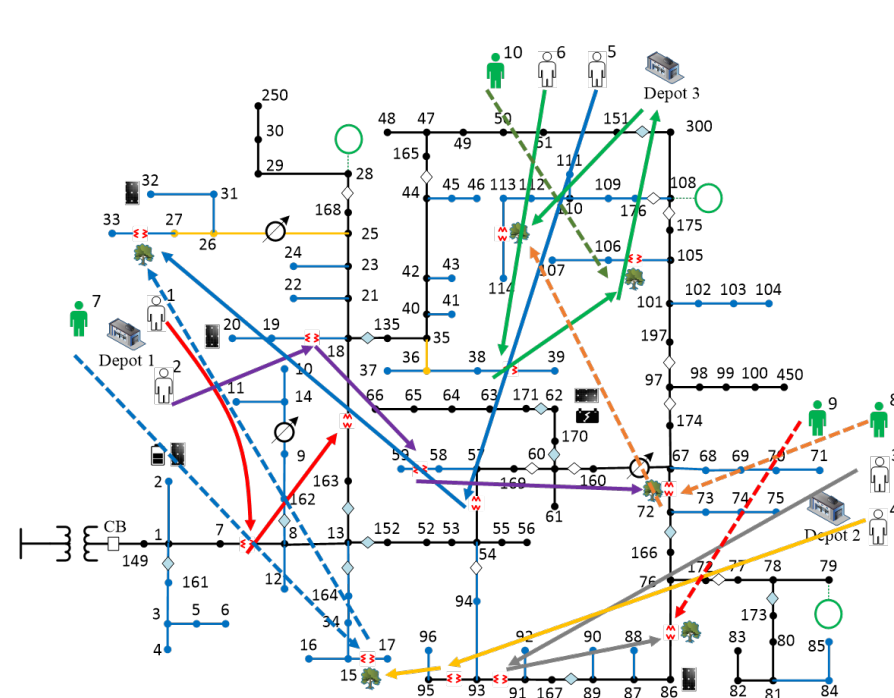
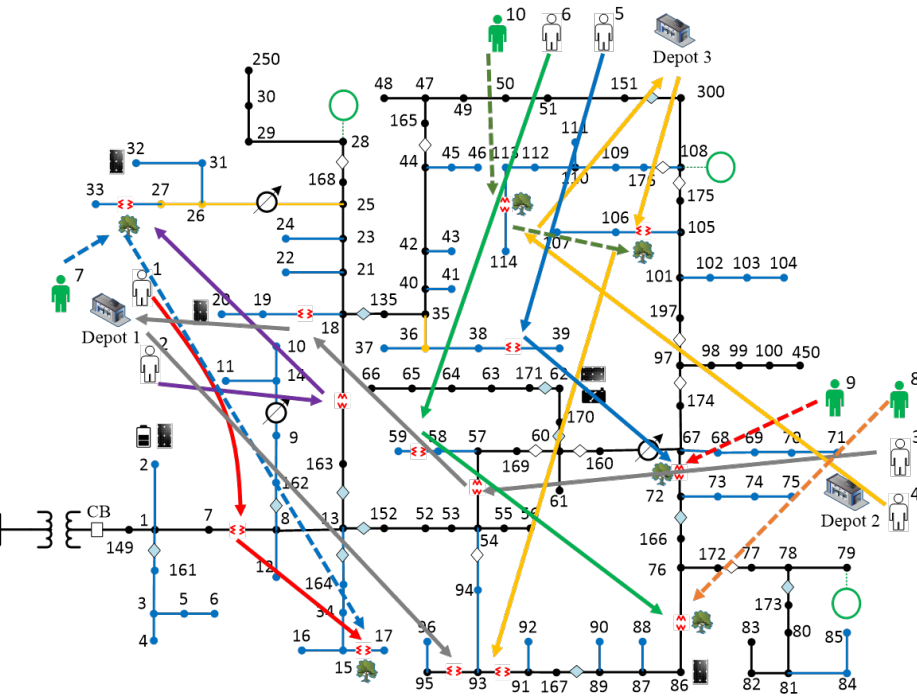
- Optimal solution is obtained by using the Reoptimization solution to warm-start CPLEX and solve the complete method



Results: Route Comparison

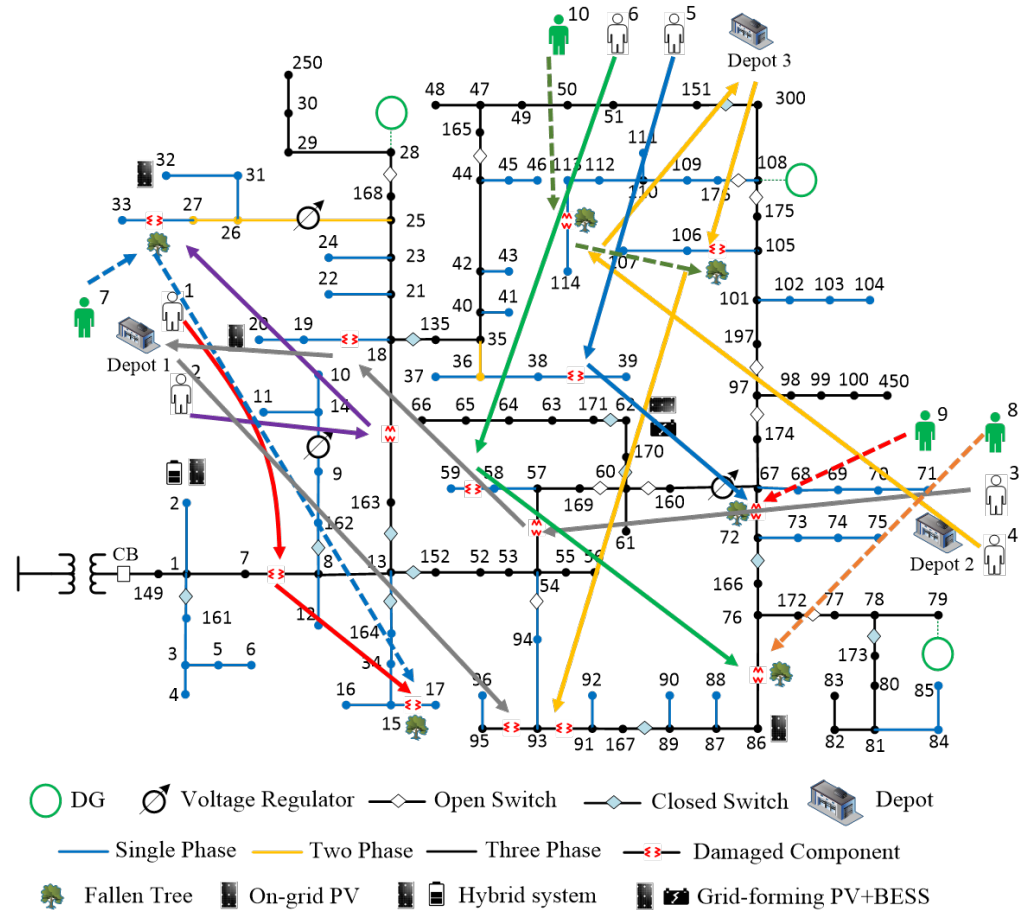
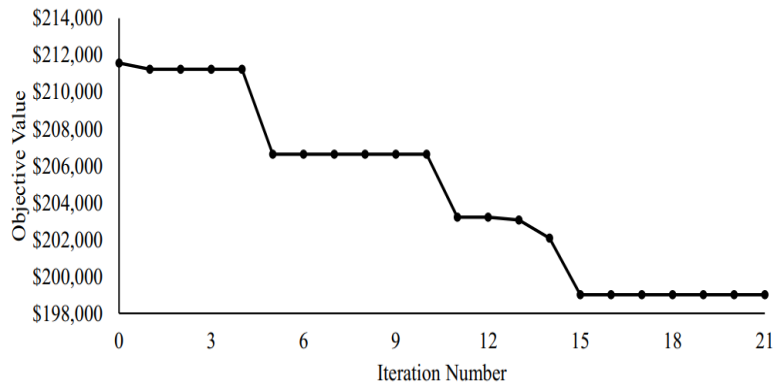
Reoptimization (optimal) route

Priority-based route



Results: Reoptimization

- Objective value: \$199,210
- Iterations: 21
- Computation time: 694 seconds



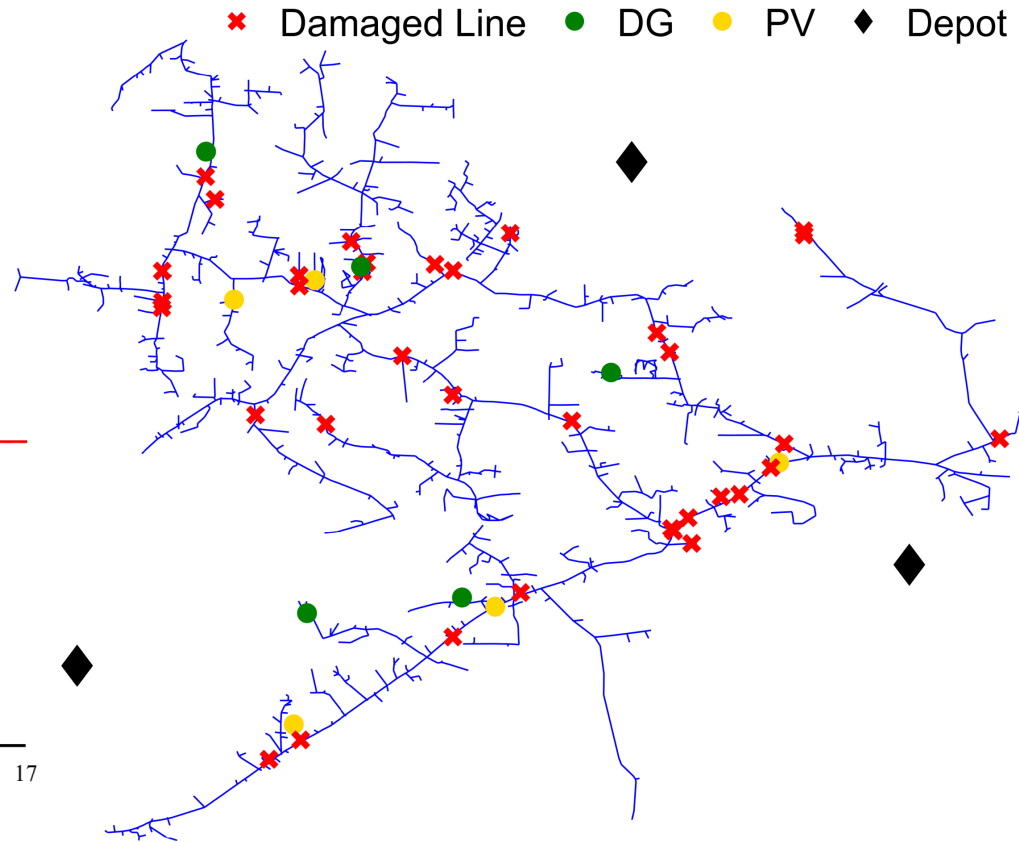
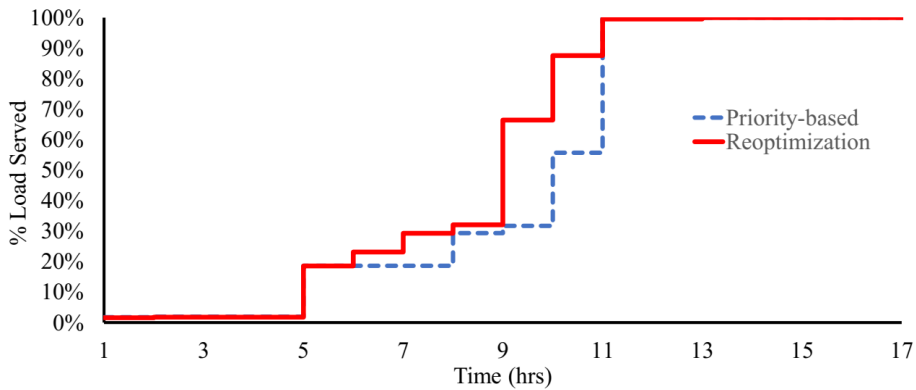
Results: DGs and Switches

- To show the importance of DGs and automatic switches, we vary the number of DGs and switches for the 14 damage case
- The best performance is obtained with the highest number of DGs and switches, as expected
- Switches are needed so that the DGs reach their full potential

Number of DGs	Number of Switches		
	23	14	7
11	\$ 199,210	\$ 231,803	\$ 363,989
4	\$ 223,291	\$ 289,435	\$ 388,111
0	\$ 345,175	\$ 412,998	\$ 421,692

Test Case: IEEE 8500-bus

- Reoptimization: \$763,184
- Priority-based: \$849,842

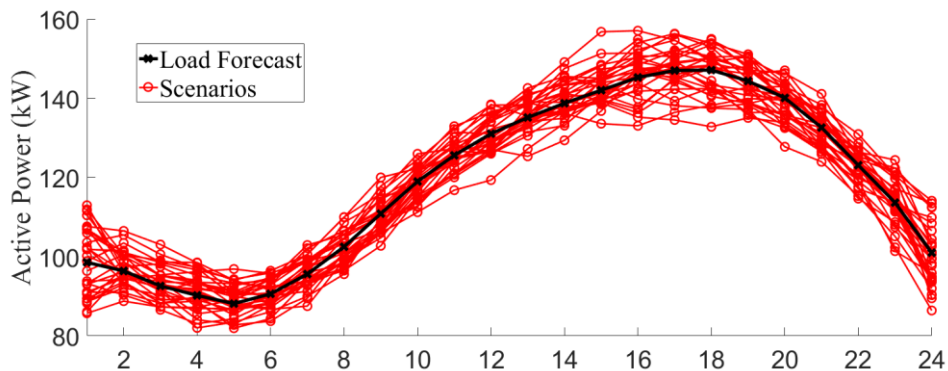


Method 2: Two-Stage Stochastic MILP

- Uncertainty
 - Repair time (Zhu 2012)
 - Demand (Lu 2013)
 - Solar irradiance (Torquato 2014)
- Objective
 - Minimize cost of shedding loads and switching operation
- First-stage constraints
 - Dispatch repair crews
 - Equipment constraints
- Second-stage constraints
 - Distribution system operation
 - Arrival time constraints
 - Connect crews routing and power operation

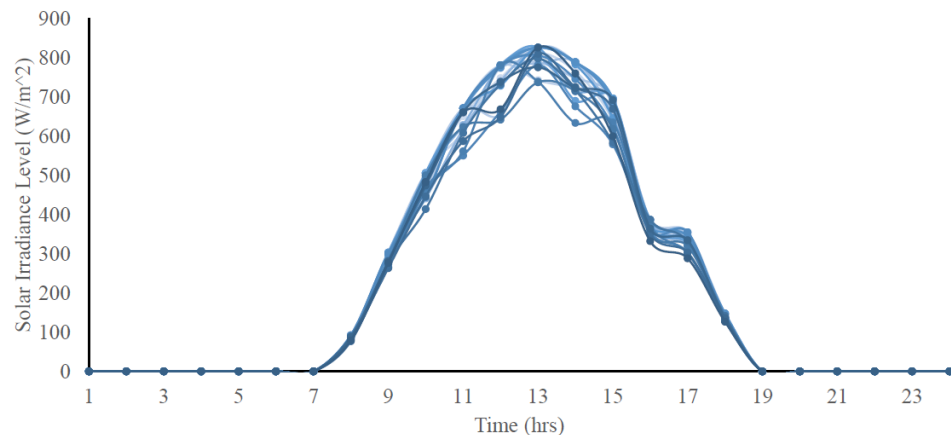
Uncertainty

- Repair time: lognormal distribution (Zhu 2012)
- Demand: truncated normal forecast error distribution (Lu 2013)
- Solar irradiance: cloud coverage level and normal distribution (Torquato 2014)



Repair time

Damage	Scenario 1	Scenario 2	Scenario 3	...	Scenario S
Line 1	2.71	3.61	1.97	...	3.11
Line 2	4.01	2.36	3.85	...	5.11
Line 3	1.24	3.21	1.06	...	4.62
Line 4	1.5	1.87	2.88	...	3.45
.
.
.
Line D	1.68	1.84	4.69	...	2.46



Stochastic DSRRP

$$\min \sum_{\forall s} \Pr(s) \sum_{\forall t} \left(\sum_{\forall \varphi} \sum_{\forall i} (1 - y_{i,t,s}) \rho_i^D P_{i,\varphi,t,s}^D + \rho^{SW} \sum_{k \in \Omega_{SW}} \gamma_{k,t,s} \right)$$

$$\sum_{\forall m \in N} x_{\phi_c^0, m, c} = 1, \forall c$$

$$\sum_{\forall m \in N} x_{m, \phi_c^1, c} = 1, \forall c$$

$$\sum_{\forall n \in N \setminus \{m\}} x_{m,n,c} - \sum_{\forall n \in N \setminus \{m\}} x_{n,m,c} = 0, \forall c, m \in N \setminus \{\phi_c^0, \phi_c^1\}$$

$$\sum_{\forall c \in C^L} \sum_{\forall m \in N \setminus \{n\}} x_{m,n,c} = 1, \forall n \in \Omega_{DL}$$

$$\sum_{\forall c \in C^T} \sum_{\forall m \in N \setminus \{n\}} x_{m,n,c} = 1, \forall n \in \Omega_{DT}$$

$$E_{d,\tau}^D \geq \sum_{\forall c \in C^L, \phi_c^d = d} E_{c,\phi_c^d, \tau}^C + \sum_{\forall c \in C^L} E_{c,d,\tau}^C, \forall d, \tau$$

$$\sum_{\forall \tau} \text{Cap}_{\tau}^R C E_{c,m,\tau} \leq \text{Cap}_{\tau}^C, \forall m, c \in C^L$$

$$\sum_{\forall n \in N} x_{n,m,c} \mathcal{R}_{m,\tau} \leq C E_{c,m,\tau}, \forall m, \tau, c \in C^L$$

$$-M(1 - x_{m,n,c}) \leq C E_{c,m,\tau} - \mathcal{R}_{m,\tau} - C E_{c,n,\tau} \leq M(1 - x_{m,n,c}),$$

$$\forall m \in N \setminus \{\phi_c^1\}, n \in N \setminus \{\phi_c^0, m\}, c \in C^L, \tau$$

$$-M(1 - x_{d,n,c}) \leq C E_{c,d,\tau} + E_{c,d,\tau}^C - C E_{c,n,\tau} \leq M(1 - x_{d,n,c}),$$

$$\forall d, n \in N \setminus \{\phi_c^0, \phi_c^1, d\}, c \in C^L, \tau$$

$$-M(1 - x_{\phi_c^0, n, c}) \leq E_{c,\phi_c^0, \tau}^C - C E_{c,n,\tau} \leq M(1 - x_{\phi_c^0, n, c}), \forall n \in N \setminus \{\phi_c^0\}, c \in C^L, \tau$$

$$P_{i,\varphi,t}^L = y_{i,t} P_{i,\varphi,t}^D + (y_{i,t} - y_{i,\max(t-\lambda,0)}) P_{i,\varphi,t}^U, \forall i, \varphi, t, s$$

$$Q_{i,\varphi,t,s}^L = y_{i,t,s} Q_{i,\varphi,t,s}^D + (y_{i,t,s} - y_{i,\max(t-\lambda,0,s)}) Q_{i,\varphi,t,s}^U, \forall i, \varphi, t, s$$

$$y_{i,t+1,s} \geq y_{i,t,s}, \forall i, t, s$$

$$0 \leq P_{i,\varphi,t,s}^G \leq P_i^{Gmax}, \forall i, \varphi, t, s$$

$$0 \leq Q_{i,\varphi,t,s}^G \leq Q_i^{Gmax}, \forall i, \varphi, t, s$$

$$-u_{k,t,s} p_{k,\varphi} P_k^{Kmax} \leq P_{k,\varphi,t,s}^K \leq u_{k,t,s} p_{k,\varphi} P_k^{Kmax}, \forall k, \varphi, t, s$$

$$-u_{k,t,s} p_{k,\varphi} Q_k^{Kmax} \leq Q_{k,\varphi,t,s}^K \leq u_{k,t,s} p_{k,\varphi} Q_k^{Kmax}, \forall k, \varphi, t, s$$

$$\sum_{\forall k \in K(i,.)} P_{k,\varphi,t,s}^K + P_{i,\varphi,t,s}^G + P_{i,\varphi,t,s}^{PV} + P_{i,\varphi,t,s}^{dch} = \sum_{\forall k \in K(i,.)} P_{k,\varphi,t,s}^K + P_{i,\varphi,t,s}^L + P_{i,\varphi,t,s}^{ch}, \forall i, \varphi, t, s$$

$$\sum_{\forall k \in K(i,.)} Q_{k,\varphi,t,s}^K + Q_{i,\varphi,t,s}^G + Q_{i,\varphi,t,s}^{PV} + Q_{i,\varphi,t,s}^{ES} + u_{i,t,s}^C Q_{i,\varphi}^C = \sum_{\forall k \in K(i,.)} Q_{k,\varphi,t,s}^K + Q_{i,\varphi,t,s}^L, \forall i, \varphi, t, s$$

$$U_{j,t,s} - U_{i,t,s} + \bar{Z}_k S_{k,s}^* + \bar{Z}_k^* S_{k,s} \leq (2 - u_{k,t,s} - p_k) M, \forall k \in \Omega_L \setminus \Omega_V, t, s$$

$$U_{j,t,s} - U_{i,t,s} + \bar{Z}_k S_{k,s}^* + \bar{Z}_k^* S_{k,s} \geq -(2 - u_{k,t,s} - p_k) M, \forall k \in \Omega_L \setminus \Omega_V, t, s$$

$$(0.9)^2 U_{i,\varphi,t,s} \leq U_{j,\varphi,t,s} \leq (1.1)^2 U_{i,\varphi,t,s}, \forall k \in \Omega_V, \varphi, t, s$$

$$\mathcal{X}_{i,t,s} U_{min} \leq U_{i,t,s} \leq \mathcal{X}_{i,t,s} U_{max}, \forall i, t, s$$

$$2u_{k,t,s} \geq \mathcal{X}_{i,t,s} + \mathcal{X}_{j,t,s}, \forall k \in \Omega_{DL}, t, s$$

$$u_{k,t,s} = 1, \forall k \notin \{\Omega_{SW} \cup \Omega_{DL}\}, t, s$$

$$\gamma_{k,t,s} \geq u_{k,t,s} - u_{k,t-1,s}, \forall k \in \Omega_{SW}, t, s$$

$$\gamma_{k,t,s} \geq u_{k,t-1,s} - u_{k,t,s}, \forall k \in \Omega_{SW}, t, s$$

$$\sum_{k \in \Omega_K(i)} u_{k,t,s} \leq |\Omega_K(i)| - 1, \forall i, t, s$$

$$P_{i,\varphi,t,s}^{PV} = \frac{I r_{i,t,s}}{(1000W/m^2)} \bar{P}_i^{PV}, \forall i \in \Omega_{PV} \setminus \Omega_{PV}^G, \varphi, t, s$$

$$P_{i,\varphi,t,s}^{PV} = \mathcal{X}_{i,t,s} \frac{I r_{i,t,s}}{(1000W/m^2)} \bar{P}_i^{PV}, \forall i \in \Omega_{PV}^G, \varphi, t, s$$

$$|Q_{i,\varphi,t,s}^{PV}| \leq \sqrt{(S_i^{PV})^2 - (\hat{P}_{i,t,s}^{PV})^2}, \forall i \in \Omega_{PV} \setminus \Omega_{PV}^G, \varphi, t, s$$

$$|Q_{i,\varphi,t,s}^{PV}| \leq \mathcal{X}_{i,t,s} \sqrt{(S_i^{PV})^2 - (\hat{P}_{i,t,s}^{PV})^2}, \forall i \in \Omega_{PV}^G, \varphi, t, s$$

$$\text{where } \hat{P}_{i,t,s}^{PV} = \frac{I r_{i,t,s}}{(1000W/m^2)} \bar{P}_i^{PV}$$

$$v_{i,\varphi,t,s}^S + \sum_{k \in K(i,s)} v_{k,\varphi,t,s}^f = \mathcal{X}_{i,t,s} + \sum_{k \in K(i,.)} v_{k,\varphi,t,s}^f, \forall i, \varphi, t, s$$

$$\sum_{\forall \varphi} \sum_{\forall t} v_{i,\varphi,t,s}^S = 0, \forall i \in \Omega_B \setminus \{\Omega_{PV}^G \cup \Omega_G \cup \Omega_{Sub}\}, s$$

$$-(u_{k,t,s} p_{k,\varphi}) M \leq v_{i,\varphi,t,s}^f \leq (u_{k,t,s} p_{k,\varphi}) M, \forall k \in \Omega_K, \varphi, t, s$$

$$\mathcal{X}_{i,t,s} \geq y_{i,t,s}, \forall i \in \Omega_B \setminus \{\Omega_G \cup \Omega_{PV}^G \cup \Omega_{PV}^H\}, t, s$$

$$0 \leq P_{i,\varphi,t,s}^{ch} \leq u_{i,t,s}^E \bar{P}_i^{ch}, \forall i \in \Omega_{ES}, \varphi, t, s$$

$$0 \leq P_{i,\varphi,t,s}^{dch} \leq (1 - u_{i,t,s}^E) \bar{P}_i^{dch}, \forall i \in \Omega_{ES}, \varphi, t, s$$

$$E_{i,t,s}^S = E_{i,t-1,s}^S + \Delta t (\eta c \sum_{\forall \varphi} P_{i,\varphi,t,s}^{ch} - \frac{\sum_{\forall \varphi} P_{i,\varphi,t,s}^{dch}}{\eta d}), \forall i \in \Omega_{ES}, t, s$$

$$\underline{E}_i^S \leq E_{i,t,s}^S \leq \bar{E}_i^S, \forall i \in \Omega_{ES}, t, s$$

$$-u_{i,t,s}^E S_i^{ES} \leq Q_{i,\varphi,t,s}^{ES} \leq u_{i,t,s}^E S_i^{ES}, \forall i \in \Omega_{ES}, \varphi, t, s$$

$$|(P_{i,\varphi,t,s}^{ch} + P_{i,\varphi,t,s}^{dch}) + Q_{i,\varphi,t,s}^{ES}| \leq \sqrt{2} S_i^{ES}, \forall i \in \Omega_{ES}, \varphi, t, s$$

$$|(I_{i,\varphi,t,s}^{ch} + P_{i,\varphi,t,s}^{dch}) - Q_{i,\varphi,t,s}^{ES}| \leq \sqrt{2} S_i^{ES}, \forall i \in \Omega_{ES}, \varphi, t, s$$

$$\alpha_{m,c,s} + E T_{m,c,s} + t r_{m,n} - (1 - x_{m,n,c}) M \leq \alpha_{m,c,s}, \forall m \in N \setminus \{\phi_c^1\}, n \in N \setminus \{\phi_c^0, m\}, c, s$$

$$\sum_{c \in C^L} \alpha_{m,c,s} \geq \sum_{c \in C^T} \alpha_{m,c,s} + E T_{m,c,s}, \forall m \in N, s$$

$$\sum_{\forall t} f_{m,t,s} = 1, \forall m \in \Omega_{DL}, s$$

$$\sum_{\forall t} t f_{m,t,s} \geq \sum_{\forall c} (\alpha_{m,c,s} + E T_{m,c,s}) \sum_{\forall n \in N} x_{m,n,c}, \forall m \in \Omega_{DL}, s$$

$$0 \leq \alpha_{m,c,s} \leq M \sum_{n \in N} x_{m,n,c}, \forall m \in N \setminus \{\phi_c^0, \phi_c^1\}, c, s$$

$$u_{m,t,s} = \sum_{\bar{t}=1}^t f_{m,\bar{t},s}, \forall m \in \Omega_{DL}, t, s$$

Stochastic vs Deterministic

- Solve the 14 damaged lines test case using:
 - Stochastic method (DS-DSRRP) → consider different scenarios ξ → obtain route x^S
 - Static-Reoptimization → consider average scenario → obtain route x^R
 - Dynamic-Reoptimization → consider average scenario → obtain route x^D → update if the repair time changes
- Set the routes as constant (not for the dynamic method)
- Generate a new scenario ξ_{case} for the repair times to be the actual realization and calculate the objective value

The objective value for the IEEE 123-bus system (14 damaged lines) with constant routing solutions and different scenario realizations

Case	DS-DSRRP (PH)		Static-Reoptimization		Dynamic-Reoptimization		Optimal
	$F(x^S, \xi_{case})$	% Gap	$F(x^R, \xi_{case})$	% Gap	$F(x^D, \xi_{case})$	% Gap	
1	\$256,104.7	10.84%	\$241,661.9	4.59%	\$232,728.8	0.72%	\$231,065.4
2	\$248,671.7	6.98%	\$299,586.4	28.88%	\$245,558.6	5.64%	\$232,447.7
3	\$269,505.5	6.85%	\$291,036.7	15.38%	\$259,189.3	2.76%	\$252,235.3
4	\$251,256.7	13.27%	\$268,590.5	21.08%	\$236,415.2	6.58%	\$221,828.2
5	\$240,549.3	15.22%	\$246,431.5	18.04%	\$221,790.7	6.24%	\$208,772.2

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Conclusions

- Effective preparation procedures can ensure that enough equipment is present for repairing the damaged components in the network and facilitate a faster restoration process
- Co-optimizing repair and recovery operation leads to better results compared to solving the two problems separately
- Efficient repair schedule along with DGs and controllable switches limit the outage size and can decrease the restoration time
- Advanced solution algorithms are required for solving the co-optimization problem due to its complexity
- A dynamic approach where the deterministic solution is periodically updated can achieve better solutions than stochastic programming

Publications

Journal Papers

1. A. Arif, Z. Wang, J. Wang, and C. Chen, "Repair and resource scheduling in unbalanced distribution systems using neighborhood search," *IEEE Trans. Smart Grid*, Under Review.
2. A. Arif, Z. Wang, C. Chen, B. Chen, "A stochastic multi-commodity logistic model for disaster preparation in distribution systems," *IEEE Trans. Smart Grid*, to be published.
3. A. Arif, S. Ma, Z. Wang, J. Wang, S. M. Ryan and C. Chen, "Optimizing service restoration in distribution systems with uncertain repair time and demand," *IEEE Trans. Power Syst.*, vol. 33, no. 6, pp. 6828-6838, Nov. 2018.
4. A. Arif, Z. Wang, J. Wang, B. Mather, H. Bashualdo, and D. Zhao, "Load modeling - a review," *IEEE Trans. Smart Grid*, vol. 9, no. 6, pp. 5986-5999, Nov. 2018.
5. A. Arif, Z. Wang, J. Wang, and C. Chen, "Power distribution system outage management with co-optimization of repairs, reconfiguration, and DG dispatch," *IEEE Trans. Smart Grid*, vol. 9, no. 5, pp. 4109-4118, Sept. 2018.
6. A. Arif and Z. Wang, "Networked microgrids for service restoration in resilient distribution systems," *IET Gener. Transm. Distrib.*, vol. 11, no. 14, pp. 3612-3619, Sep. 2017.

Conference Paper

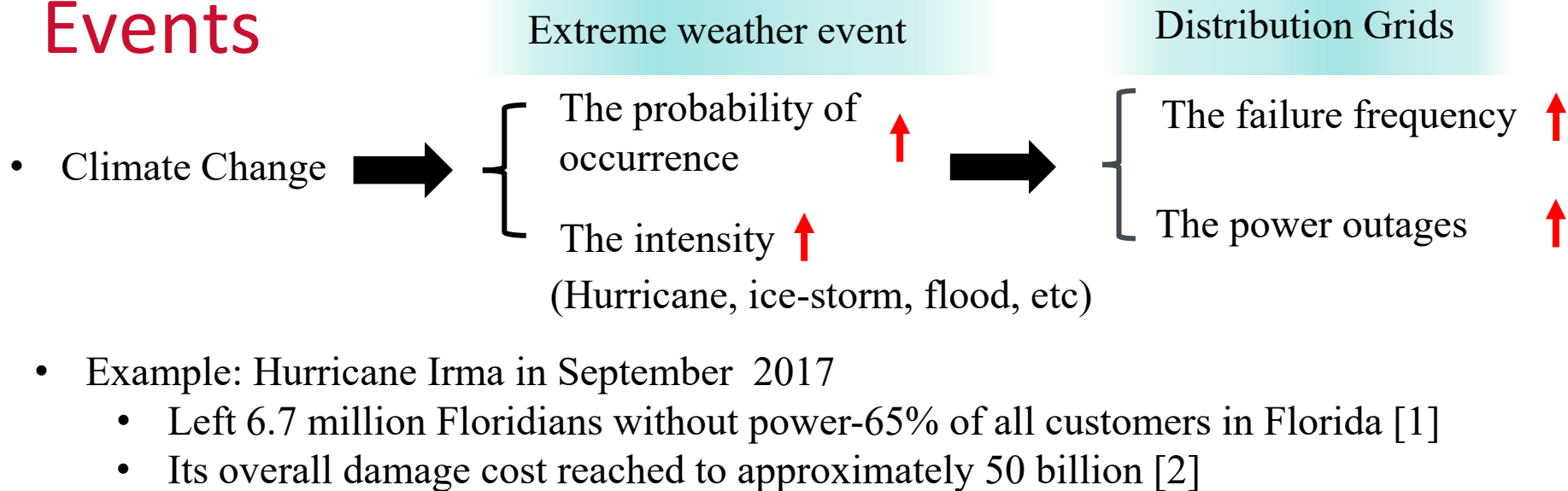
1. S. Ma, S. Li, Z. Wang, A. Arif, K. Ma., "A novel MILP formulation for fault isolation and network reconfiguration in active distribution systems," *IEEE PES General Meeting*, Portland, OR, 2018, pp. 1-5.
2. A. Arif, Z. Wang, "Distribution network outage data analysis and repair time prediction using deep learning," *IEEE Int Conf. Probabilistic Methods Appl. Power Syst.*, Boise, ID, 2018, pp. 1-6.
3. A. Arif, S. Ma, Z. Wang, "Dynamic reconfiguration and fault isolation for a selfhealing distribution system," *IEEE PES Transm. Distrib. Conf. Expo.*, Denver, CO, 2018, pp. 1-5.73
4. A. Arif, S. Ma, Z. Wang, "Online decomposed optimal outage management after natural disasters," *IEEE PES General Meeting*, Chicago, IL, 2017, pp. 1-5.
5. A. Arif, S. Ma, Z. Wang, "Optimization of transmission system repair and restoration with crew routing," *IEEE North Amer. Power Symp.*, 2016, Denver, CO, pp. 1-6.
6. A. Arif and Z. Wang, "Service restoration in resilient power distribution systems with networked microgrid," *IEEE PES General Meeting*, Boston, MA, 2016, pp. 1-5.

Part III Resilience-oriented Long-term Planning

Outline

- Motivation and Introduction
- Problem Statement
- Literature Review
- Research Objectives and Contributions
- Stochastic Decision Process
- Mathematical Formulation
- Solution Algorithms
- Simulation and Results
- Conclusions and Future Work

Motivation: Impacts of Extreme Weather Events



Motivation: Current Situation of Distribution Systems

- Most existing distribution systems are designed and maintained for normal weather conditions
- The classic reliability principles cannot guarantee the lights on under extreme weather events
- U.S. power grids are now old and outdated
- Utilities upgrade grids based on experiences, patrols, and observations

As power engineers, how can we improve grid resilience to survive from extreme weather events?

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Introduction: The Resilience of Distribution System

- A distribution system is considered to be *resilient* if it is able to **anticipate**, **absorb**, **adapt to**, and/or **rapidly recover** from a disruptive event [6].

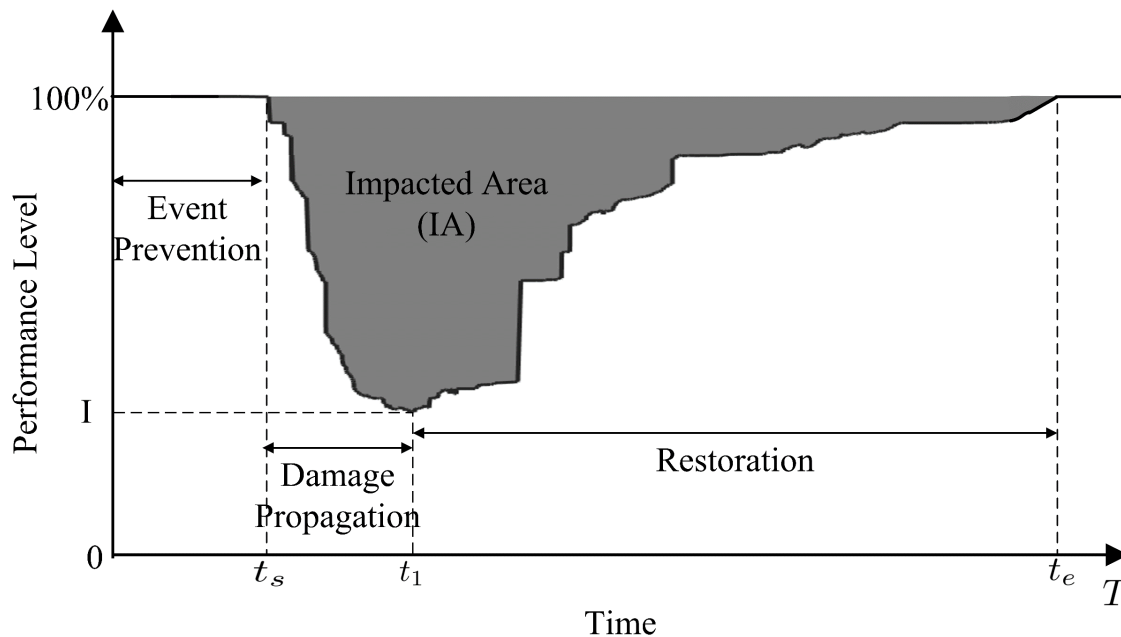


Fig.1. A general system performance curve of a distribution system following an extreme weather event

- Event prevention stage: Resistant capability
- Damage propagation stage: Absorptive and adaptive capacity
- Restoration stage: Recovery capability

Introduction: The Resilience Enhancement Measures

- Two resilience goals of distribution systems [7]:
 - System adaptation (to reduce the impact of future events)
 - System survivability (to maintain an adequate functionality during and after the event)
- Resilience enhancement measures:

Resilience-Oriented Design (ROD) Measures

- Topological and structural upgrades of the utility's infrastructures
 - Upgrading distribution poles to stronger class
 - Installing automatic switches
 - Installing back-up distributed generators (DG)

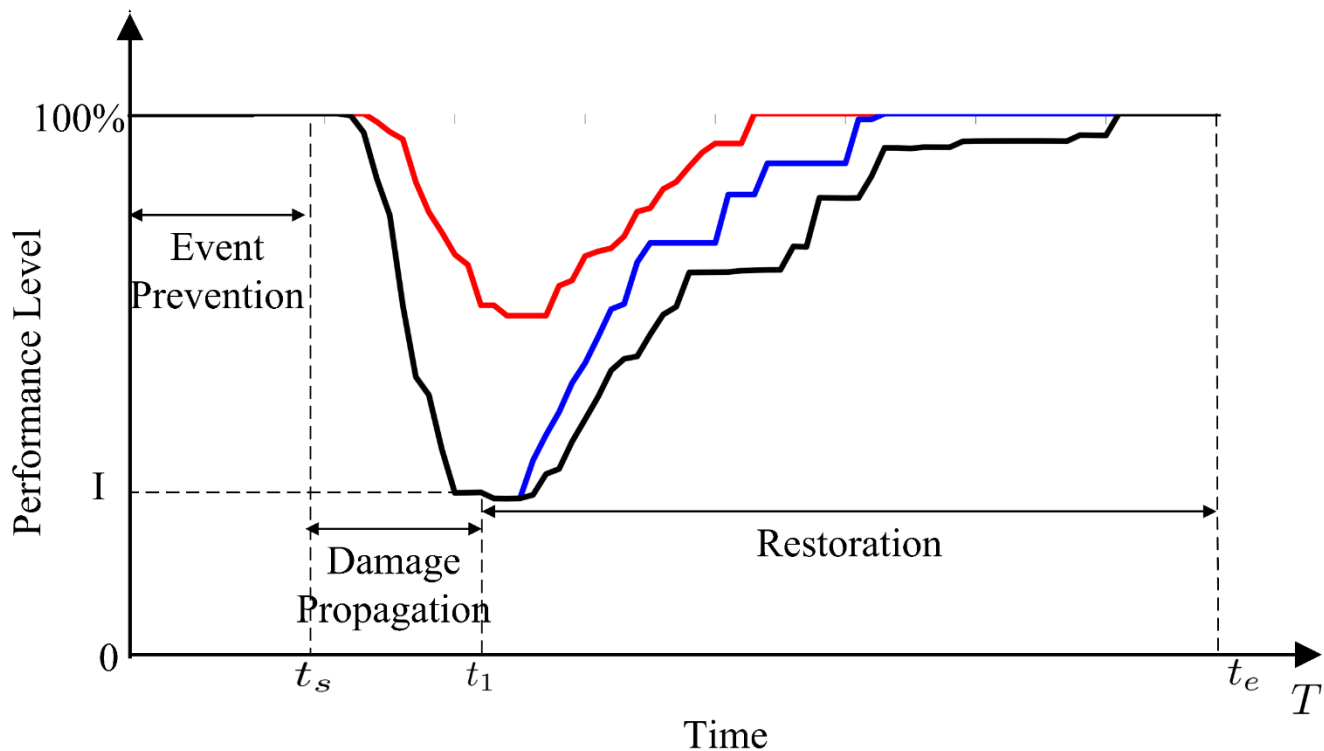


Resilience-Oriented Operational (ROO) Measures

- “Smart” control-based actions
 - Network reconfiguration
 - DG rescheduling
 - Conservation voltage regulation
 - Defensive islanding
 - Microgrid-assisted control actions
 - Priority-based load shedding

Introduction: The Big Picture

- A resilient distribution system
 - **Planning:** pole hardening, and DG and switch installation
 - **Operation:** co-optimization of repair scheduling and restoration operation



- We focus on exploring *effects of ROD measures* on *system resilience* with the consideration of operation response

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Problem Statement

- How to optimally apply ROD measures to prevent distribution system from extensive damages caused by extreme weather events
- Some *spatial-temporal correlations* exist among ROD decisions, extreme weather events, and system operations
 - Occurrence, intensity and traveling path of events are *uncertain*
 - Physical infrastructure damage status are affected by both extreme weather event and ROD decisions
 - ROD decisions affect system recovery and the associated outage/repair costs
- A *time-varying interaction* exists between structural damages and electric outage propagation
- ➔ • Difficult to capture the entire *failure-recovery-cost process* of distribution systems during and after an extreme weather event.

Literature Review

Ref	Uncertainty Consideration	Measures	Model/Algorithm
[8]	<ul style="list-style-type: none"> Use a polyhedral set to represent damage uncertainty 	<ul style="list-style-type: none"> line hardening 	<ul style="list-style-type: none"> Robust optimization/column-and-constraint generation algorithm
[9]	<ul style="list-style-type: none"> Use failure probabilities of distribution lines to represent damage uncertainty set 	<ul style="list-style-type: none"> Pole hardening Vegetation management Combination of both 	<ul style="list-style-type: none"> Tri-level robust optimization/greedy algorithm
[10]	<ul style="list-style-type: none"> Use failure probabilities of overhead lines and underground gas pipelines to generate line damage uncertainty set 	<ul style="list-style-type: none"> Line hardening 	<ul style="list-style-type: none"> Tri-level robust optimization/column-and-constraint generation algorithm
[11]	<ul style="list-style-type: none"> Use fragility model to generate line damage uncertainty 	<ul style="list-style-type: none"> Line hardening DG placement Switch Installation 	<ul style="list-style-type: none"> Two-stage stochastic program/a scenario-based variable neighborhood decomposition search algorithm
[12]	<ul style="list-style-type: none"> Use fragility model to generate line damage uncertainty 	<ul style="list-style-type: none"> Line hardening (replace overhead line with underground line) MGs Networked MGs 	<ul style="list-style-type: none"> Two-stage stochastic program/a decomposition-based heuristic algorithm
[13]	<ul style="list-style-type: none"> Use fragility model to generate line damage uncertainty Model repair time uncertainty Consider load demand uncertainty 	<ul style="list-style-type: none"> Line hardening DG placement Switch Installation 	<ul style="list-style-type: none"> Two-stage stochastic program/Progressive hedging algorithm

Research Objective

- Develop a new modeling and solution methodology for the ROD of distribution systems against wind-induced extreme weather events
 - Develop a hybrid stochastic process with a deterministic casual structure to describe the spatio-temporal correlations of ROD decisions and uncertainties
 - Formulate a two-stage stochastic mixed-integer linear program (SMILP) to capture the impacts of ROD decisions and uncertainties on system's responses to extreme weather events
 - Design solution algorithm for solving the above problems.

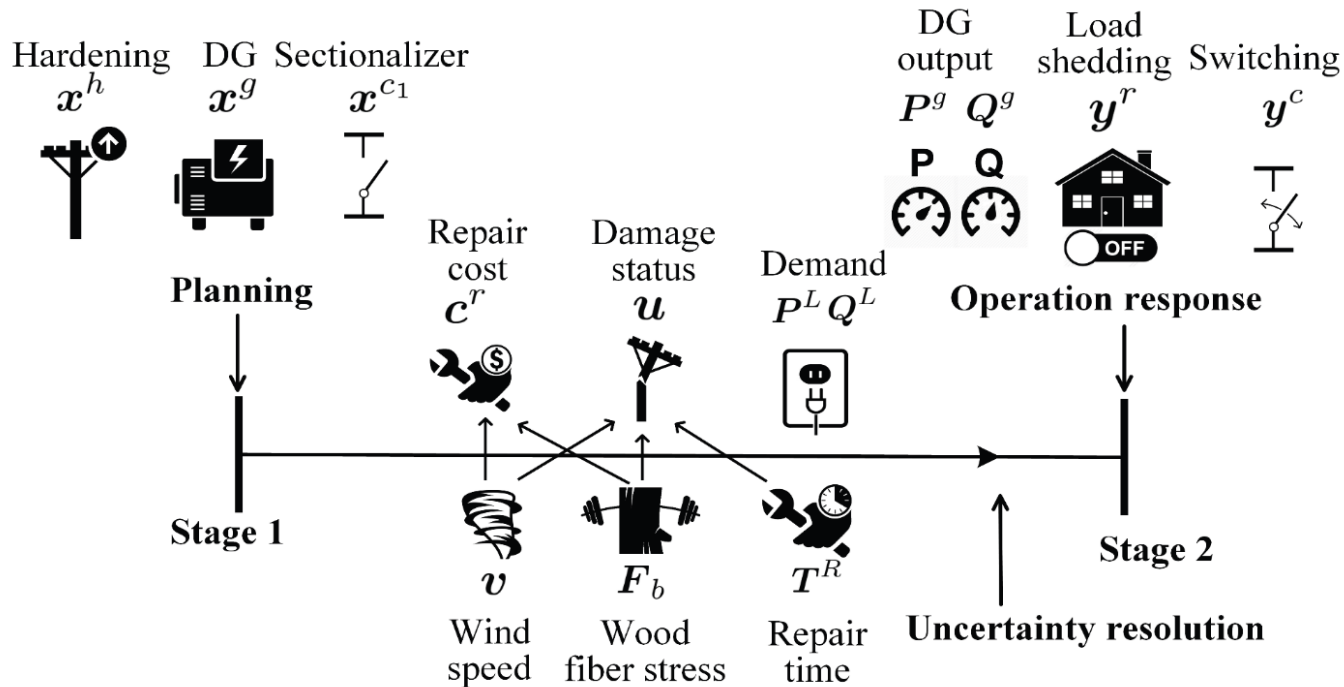
Research Contributions

- Model a hybrid independent stochastic process with a deterministic causal structure to capture the spatiotemporal correlation among the various uncertainties and ROD decisions
 - avoid establishing the high-dimension joint distribution of uncertain variables
 - model the evolving impacts of extreme weather events on physical infrastructures
- Propose a two-stage SMILP to optimally implement multiple ROD measures considering various uncertainties, thus increasing the infrastructure strength and enabling self-healing operations
 - captures the entire failure-recovery process
 - the self-healing operation in the second stage can mimic the outage propagation with minimum service interruption
- Develop a customized DD algorithm to balance optimality and solution efficiency

Stochastic Decision Process of ROD Problem

- Overview
- First-stage decisions
- Uncertainty Modeling

Overview



- ROD problem is modeled as a two-stage stochastic decision process:
 - Planner makes ROD decisions
 - The operation uncertainties are resolved during the extreme weather event
 - Operator makes the recourse decisions

First-Stage Decisions

- Hardening poles:

- Strengthening vulnerable components
- Consider 6 pole types
- Pole stress (**1 > 2 > 3 > 4 > 5 > 6**)

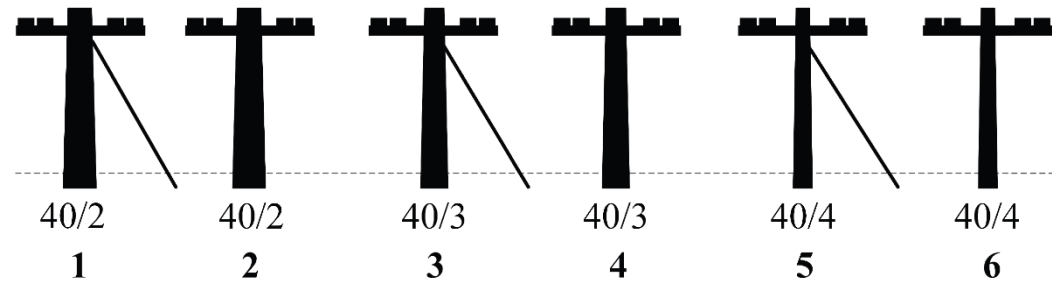


Fig.1. Pole types

- Installing Backup DGs

- Increasing adequacy of power supply

- Adding sectionalizers

- Increasing topological flexibility
- Can be added at both ends of a line

Uncertainty Modeling

- Consider three groups of random variables that have direct impacts on the evolution of the system operation state
 - Line damage status
 - Repair costs
 - Load demands

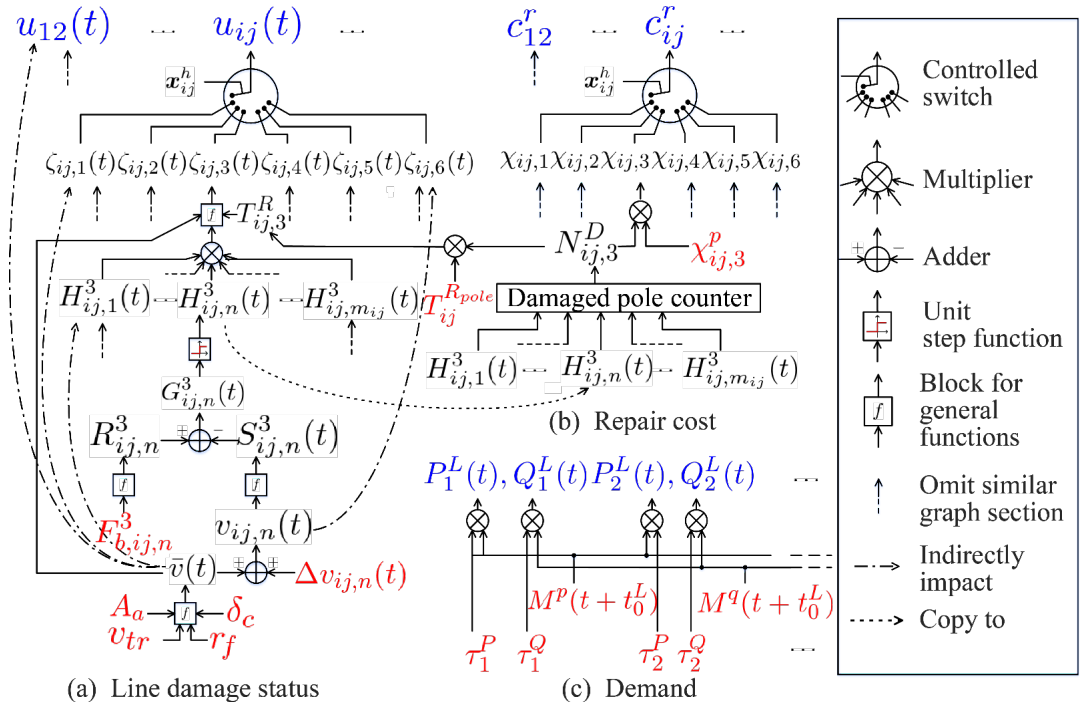
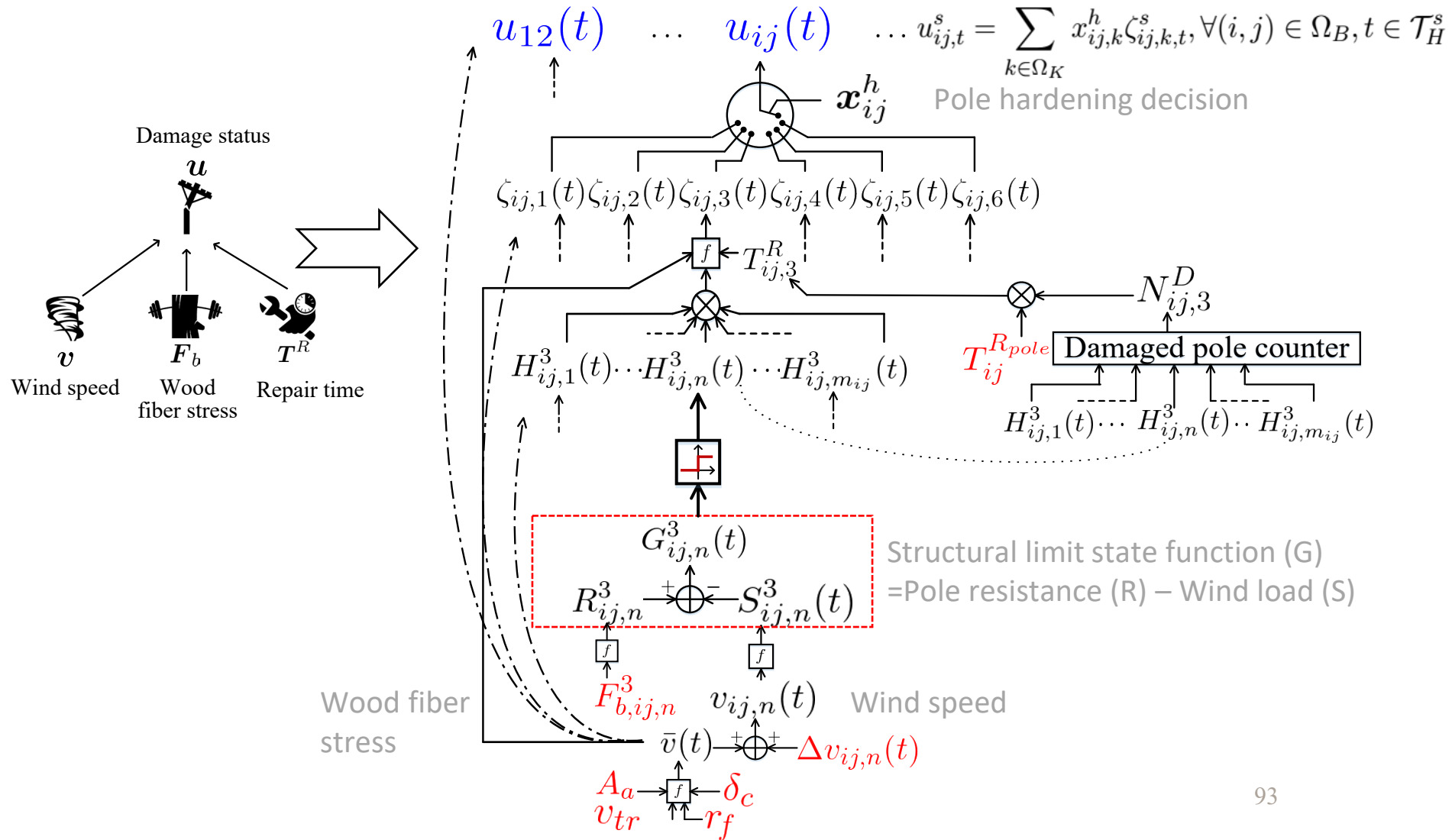
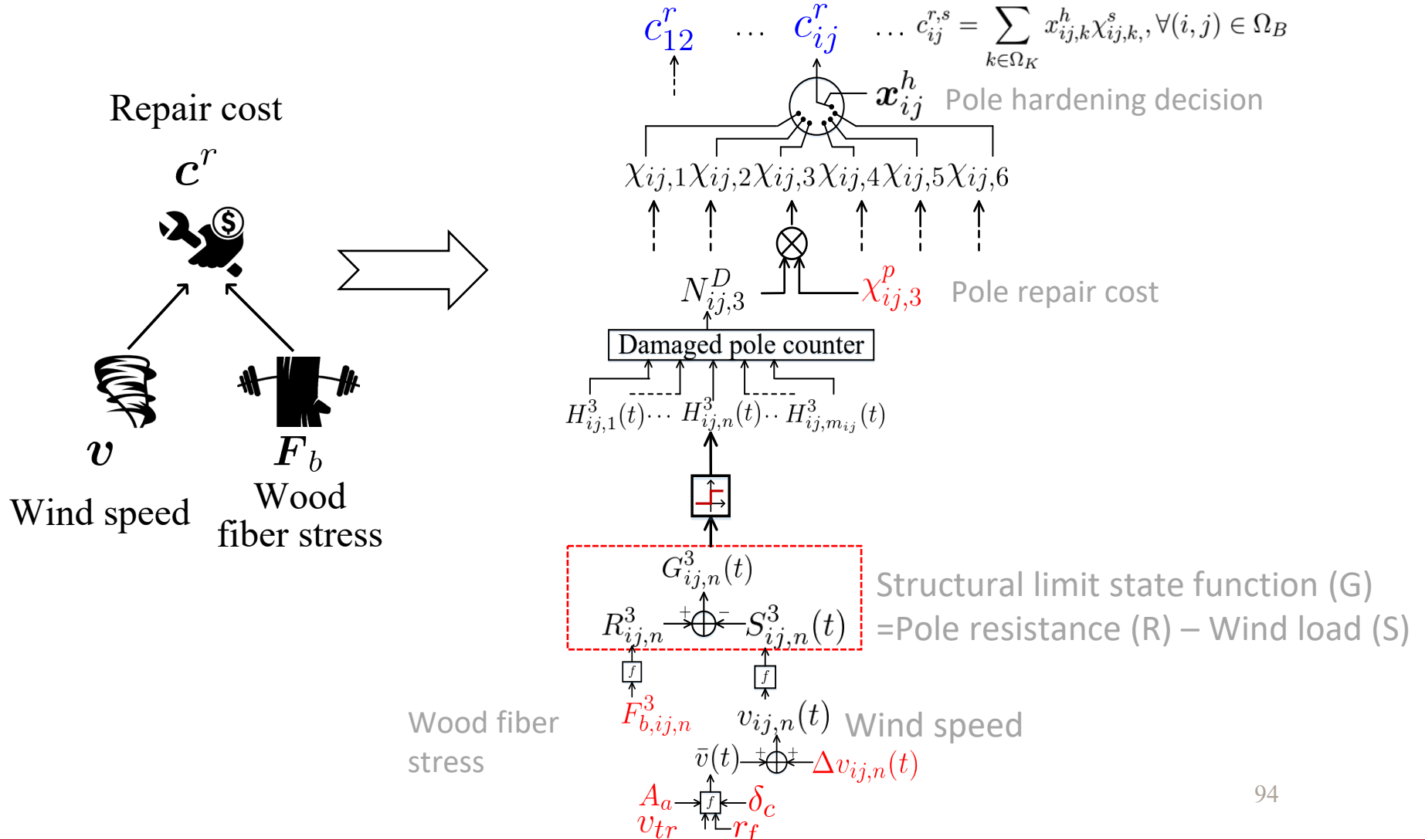


Fig.1. The structure of uncertainty space: independent observable random variables/processes (highlighted in red) + deterministic casual connections (parameterized by the first-stage decision).

(a) Line Damage Status Uncertainty



(b) Repair Cost Uncertainty



(c) Load Demand Uncertainty

$$P_i^L(t) = \tau_i^P \cdot M^P(t + t_0^L), \forall i \in \Omega_L, t \in \mathcal{T}_H$$

$$Q_i^L(t) = \tau_i^Q \cdot M^q(t + t_0^L), \forall i \in \Omega_L, t \in \mathcal{T}_H.$$

$$\tau_i^P \sim N(\bar{P}_i, (0.02\bar{P}_i)^2), \forall i \in \Omega_L$$

$$\tau_i^Q \sim N(\bar{Q}_i, (0.02\bar{Q}_i)^2), \forall i \in \Omega_L$$

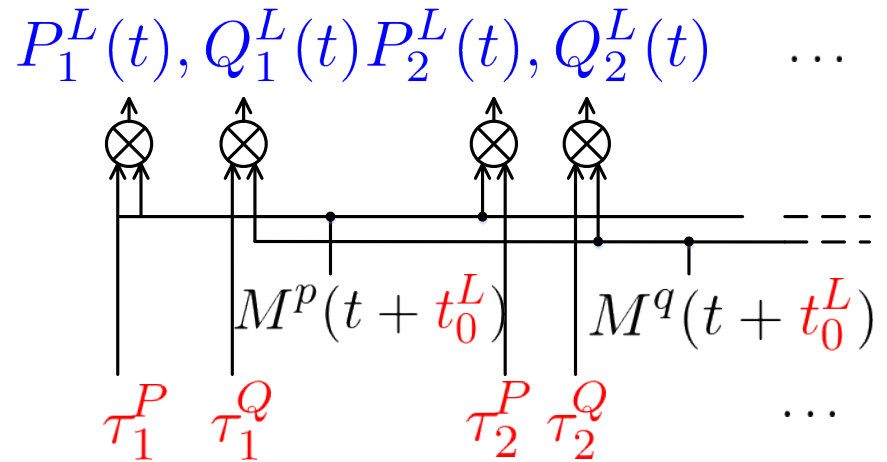


Fig.1. load demand uncertainty

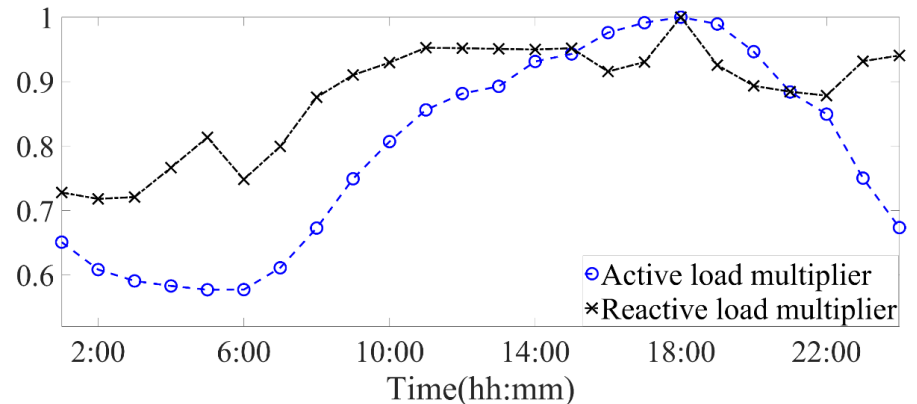
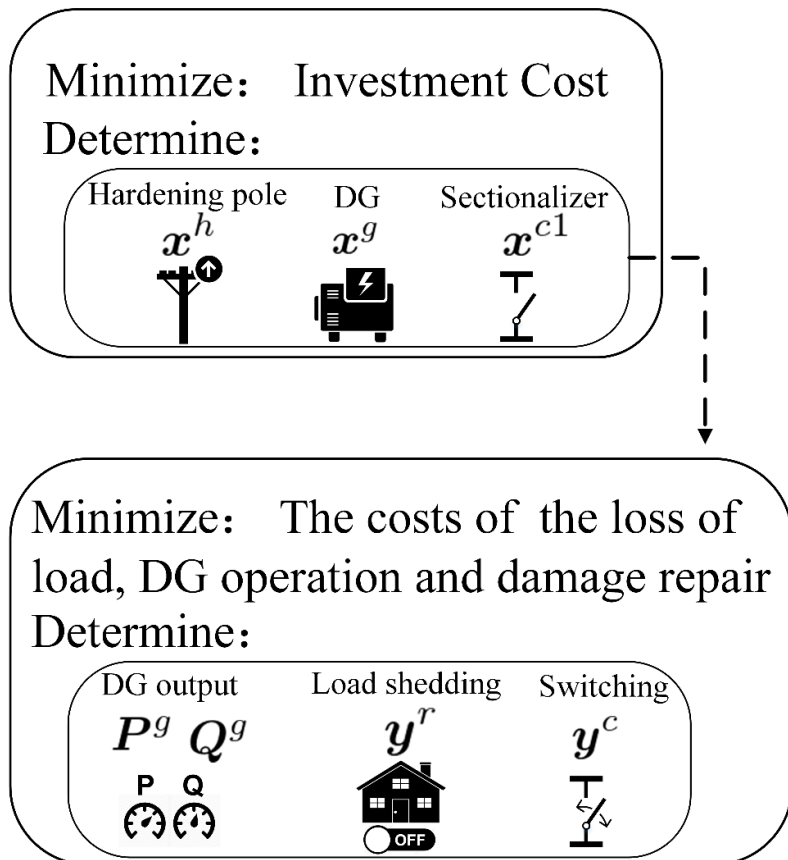


Fig.2. load profile shape at the substation (root node)

Mathematic Formulation of ROD Problem

- Overview
- First-stage Problem
- Second-stage Problem

Overview



- **Investment Stage:** identify the optimal ROD decisions
- **Operation Stage:** achieve self-healing operation
 - need a mathematic formulation to fully model power outage propagation
 - need an analytic optimization to sectionalize a distribution network into multiple self-supplied MGs while maintaining their radial network typologies

First-Stage Formulation

$$\min C_1^I(x^h) + C_1^I(x^g) + C_1^I(x^{c1}) + w_H \mathbb{E}_\xi \phi(x, \xi)$$

s.t. :

Objective:

Minimize the **ROD investment cost** and the expected cost of **the loss of load, DG operation, and damage repair** in realized extreme weather events

First stage ROD variables:

x_{ij}^h whether hardening line (i, j) (1) or not (0) $x_{ij,i}^{c1}$ whether adding a sectionalizer at the end i of line (i, j) (1) or not (0)
 x_i^g whether installing DG at node i (1) or not (0)

First stage constraints:

$$\sum_{k \in \Omega_K} x_{ij,k}^h = 1, \forall (i, j) \in \Omega_B \quad \text{Hardening strategy limit} \quad \sum_{i \in \Omega_N} x_i^g \leq N_G \quad \text{DG number limit}$$

$$x_{ij,n}^{c0} + x_{ij,n}^{c1} = x_{ij,n}^c, \forall (i, j) \in \Omega_B, n \in \{i, j\} \quad \text{Switch installation constraint}$$

$$\mathbb{E}_\xi \phi(x, \xi) \cong \sum_{s \in \mathcal{S}} p_r(s) \phi(x, s) \quad \text{The expected cost of the second stage}$$

Second-Stage Problem: Technique Outline (1)

- Model the power outage propagation (expressed by a set of constraints)

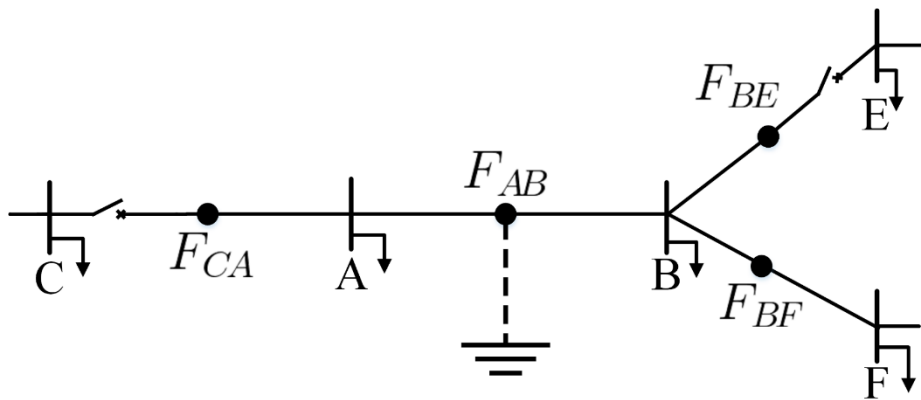


Fig.1. The illustrative example for isolating a fault

- Add a virtual node in the middle of each branch
- Apply a symmetric fault to the virtual node if the line is damaged
- Set the voltage feasible region:
$$\{0\} \cup [V^{\min}, V^{\max}]$$
- Fully curtail a load when its voltage magnitude is zero
- Set loading limits to all branches and penalize load shedding amount in the objective

Second-Stage Problem: Technique Outline (2)

- Radiality Constraints for each energized networks
 - Graph Theorem [14]: A forest of N nodes has exactly $N - N_c$ edges, where N_c is the number of connected network components.
 - How to obtain N_c in the distribution system
 - Calculate N_c by counting the degree of freedom of voltage angles
 - Formulate a virtual DC optimal power flow (VDCOPF) sub-problem to obtain this degree of freedom
 - the optimal solution of this sub-problem satisfies that the virtual loads in the same energized island are nearly equally distributed at active nodes
 - each energized island has and only has an active node with zero angle
 - The radiality constraint is satisfied *iff* the number of active branches equals the total number of active nodes minus the number of active nodes with zero angles

Second-Stage Formulation

Objective

- Minimize the cost of the loss of load, DG operation, and damage repair in a realized extreme weather event given ROD decisions

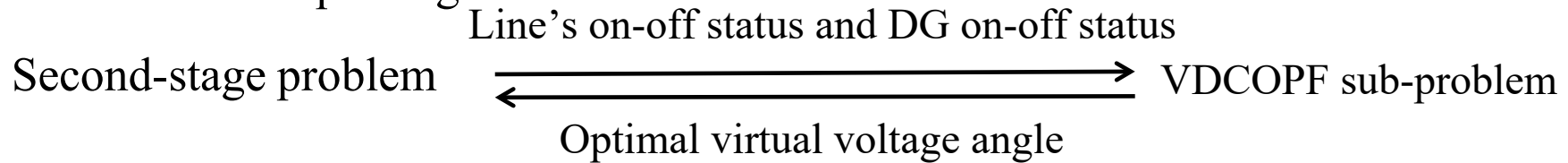
$$\phi(\mathbf{x}, s) = \min \sum_{i \in \Omega_N} \sum_{t \in \mathcal{T}_H^s} c_i^L y_{i,t}^{r,s} P_{i,t}^{L,s} \Delta t + \sum_{i \in \Omega_N} \sum_{t \in \mathcal{T}_H^s} c_i^o P_{i,t}^{g,s} \Delta t + \sum_{(i,j) \in \Omega_B} c_{ij}^{r,s}$$

Constraints

- Distribution system operation**
 - Line damage status constraint
 - Line repair cost constraint
 - Line's on-off status constraints (controlled by switch's on-off status)
 - Line flow limits (controlled by line's on-off status)
 - Linearized DistFlow equations (calculate power flow and voltage profile)
 - DG capacity limits
- Fictitious faulting logic constraints (model outage propagation)**
 - Virtual node power injection constraints
 - Voltage magnitude limits
 - Load shedding ratio limit
- The minimality condition of VDCOPF sub-problem (obtain the degree of freedom of voltage angle)**
- Zero Angle indicator constraint (indicating a node with zero angle)**

- **Key Points**

- Information passing:



- Fictitious faulting logic constraints + Distribution system operation constraints in 1)-3) + Penalty cost of load shedding in objective:

- isolate damaged lines while minimizing the de-energized network parts
- make network constraints such as power flow automatically adapt to the topology after reconfiguration

- Radiality Constraints + Zero angle indicator constraint + VDCOPF sub-problem

- can keep each energized network radial

Distribution System Operation Constraints

- 1) Line damage status constraint
- 2) Line repair cost constraint
- 3) Line's on-off status constraints
- 4) Line flow limits
- 5) Linearized DistFlow equations
- 6) DG capacity limits

$$1 \quad \left[\quad u_{ij,t}^s = \sum_{k \in \Omega_K} x_{ij,k}^h \zeta_{ij,k,t}^s, \forall (i,j) \in \Omega_B, t \in \mathcal{T}_H^s \right.$$

$$2 \quad \left[\quad c_{ij}^{r,s} = \sum_{k \in \Omega_K} x_{ij,k}^h \chi_{ij,k}^s, \forall (i,j) \in \Omega_B \right.$$

$$3 \quad \left[\quad \begin{aligned} y_{ij,t}^{c,s} &\leq x_{ij}^c, \forall (i,j) \in \Omega_{B_F}, t \in \mathcal{T}_H^s \\ x_{ij}^c + y_{ij,t}^{c,s} + 2w_{ij,t}^{o,s} &\geq 2, \forall (i,j) \in \Omega_{B_F}, t \in \mathcal{T}_H^s \\ w_{ij,t}^{o,s} + y_{ij,t}^{c,s} &\leq 1, \forall (i,j) \in \Omega_{B_F}, t \in \mathcal{T}_H^s \\ y_{ij,t}^{c,s}, w_{ij,t}^{o,s} &\in \{0, 1\}, \forall (i,j) \in \Omega_{B_F}, t \in \mathcal{T}_H^s \end{aligned} \right.$$

$$4 \quad \left[\quad \begin{array}{c} -w_{ij,t}^{o,s} \\ -w_{ij,t}^{o,s} \zeta_{ij,k,t}^s \end{array} \begin{array}{c|ccc} x_{ij}^c & y_{ij,t}^{c,s} & w_{ij,t}^{o,s} & \\ \hline 0 & 0 & 1 & \\ 0 & 1 & \text{N/A} & \end{array} \begin{array}{c} x_{ij}^c & y_{ij,t}^{c,s} & w_{ij,t}^{o,s} \\ \hline 1 & 0 & 1 \\ 1 & 1 & 0 \end{array} \in \mathcal{T}_H^s \right.$$

$$5 \quad \left[\quad \begin{aligned} &\sum_{\{j|(i,j) \in \Omega_{B_F}\}} -w_{ij,t}^{o,s} \zeta_{ij,k,t}^s \quad \text{*N/A: the case should be infeasible.} \\ &\sum_{\{j|(i,j) \in \Omega_{B_F}\}} Q_{ij,t}^s = Q_{i,t}^{g,s} - (1 - y_{i,t}^{r,s}) Q_{i,t}^L, \forall i \in \Omega_N, t \in \mathcal{T}_H^s \\ &V_{i,t}^s - \frac{R_{ij}^e P_{ij,t}^s + X_{ij}^e Q_{ij,t}^s}{V_0} - (1 - w_{ij,t}^{o,s}) M_1 \leq V_{j,t}^s \leq V_{i,t}^s - \\ &\frac{R_{ij}^e P_{ij,t}^s + X_{ij}^e Q_{ij,t}^s}{V_0} + (1 - w_{ij,t}^{o,s}) M_1, \forall i \in \Omega_{N_F}, t \in \mathcal{T}_H^s \end{aligned} \right.$$

$$6 \quad \left[\quad \begin{aligned} 0 &\leq P_{i,t}^{g,s} \leq x_i^g P_i^{g,\max}, \forall i \in \Omega_N, t \in \mathcal{T}_H^s \\ 0 &\leq Q_{i,t}^{g,s} \leq x_i^g Q_i^{g,\max}, \forall i \in \Omega_N, t \in \mathcal{T}_H^s \end{aligned} \right.$$

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Binary variables:

$u_{ij,t}^s$ Line damage status

$y_{ij,t}^{c,s}$ Sectionlizer on-off status

$w_{ij,t}^{o,s}$ Line on-off status

Fictitious Faulting Logic Constraints

- 1) Virtual node power injection constraints $\mathbf{1} \left[\begin{array}{l} -u_{ij,t}^s M_2 \leq \sum_{k \in \{i,j\}} P_{kf_{ij},t}^s + \varepsilon_1 \cdot V_{i,t}^s \leq u_{ij,t}^s M_2, \forall (i,j) \in \Omega_B, f_{ij} \in \Omega_{N_F}, t \in \mathcal{T}_H^s \\ -u_{ij,t}^s M_2 \leq \sum_{k \in \{i,j\}} Q_{kf_{ij},t}^s \leq u_{ij,t}^s M_2, \forall (i,j) \in \Omega_B, f_{ij} \in \Omega_{N_F}, t \in \mathcal{T}_H^s \end{array} \right.$
- 2) Voltage magnitude limits $\mathbf{2} \left[\begin{array}{l} w_{i,t}^{m,s} V_i^{\min} \leq V_{i,t}^s \leq w_{i,t}^{m,s} V_i^{\max}, \forall i \in \Omega_{N_F}, t \in \mathcal{T}_H^s \\ u_{ij,t}^s + w_{f_{ij},t}^{m,s} \leq 1, \forall (i,j) \in \Omega_B, f_{ij} \in \Omega_F, t \in \mathcal{T}_H^s \\ w_{i,t}^{m,s} \in \{0, 1\}, \forall i \in \Omega_{N_F}, t \in \mathcal{T}_H^s \end{array} \right.$
- 3) Load shedding ratio limit $\mathbf{3} \left[1 - w_{i,t}^{m,s} \leq y_{i,t}^{r,s} \leq 1, \forall i \in \Omega_N, t \in \mathcal{T}_H^s \right.$

Radiality constraints

- 1) Radiality constraint $\mathbf{1} \left[\sum_{(i,j) \in \Omega_{B_F}} w_{ij,t}^{b,s} = \sum_{i \in \Omega_{N_F}} w_{i,t}^{m,s} - \sum_{i \in \Omega_{N_F}} w_{i,t}^{a,s} \right.$
- 2) Active branch identification constraint $\mathbf{2} \left[\begin{array}{l} w_{ij,t}^{o,s} + w_{i,t}^{m,s} - 1 \leq w_{ij,t}^{b,s} \leq 0.5w_{ij,t}^{o,s} + 0.5w_{i,t}^{m,s}, \forall i \in \Omega_{N_F}, (i,j) \in \Omega_{B_F}, t \in \mathcal{T}_H^s \\ w_{i,t}^{a,s}, w_{ij,t}^{b,s} \in \{0, 1\}, \forall i \in \Omega_{N_F}, (i,j) \in \Omega_{B_F}, t \in \mathcal{T}_H^s \end{array} \right.$

Zero angle indicator constraint

$$w_{i,t}^{a,s} - 1 \leq \frac{1}{2|\Omega_{N_F}|} (\mu_{d,i,t}^s - 1 + \varepsilon_3) \leq w_{i,t}^{a,s}, \forall i \in \Omega_N, t \in \mathcal{T}_H^s$$

Binary variables:

$$\begin{array}{ll} w_{i,t}^{m,s} & \text{active node} \\ w_{i,t}^{a,s} & \text{node with zero voltage angle} \\ w_{i,t}^{b,s} & \text{active branch} \end{array}$$

The Minimality Condition of VDCOPF Sub-problem

- To realize that a connected network component (healthy MG) has one and only one degree of freedom of voltage angle under the condition of full DC power flow equations

$$\left(\mathcal{P}_{L,t}^{s,*}, \mathcal{P}_{l,t}^{s,*}, \theta_t^{s,*} \right) = \arg \min_{\mathcal{P}_{L,t}^s, \mathcal{P}_{l,t}^s, \theta_t^s} \left\{ \sum_{i \in \Omega_{NF}} \left(\theta_{i,t}^s + \frac{\alpha_L}{2} (\mathcal{P}_{L,i,t}^s)^2 \right) \right.$$

$$\left. \begin{array}{l} a: -(1 - w_{ij,t}^{o,s}) M_3 \leq \mathcal{P}_{ij,t}^s - S_0 B'_{ij} (\theta_{i,t}^s - \theta_{j,t}^s) \leq (1 - w_{ij,t}^{o,s}) M_3, \forall (i,j) \in \Omega_{BF} \\ b: -w_{ij,t}^{o,s} M_3 \leq \mathcal{P}_{ij,t}^s \leq w_{ij,t}^{o,s} M_3, \forall (i,j) \in \Omega_{BF} \\ c: \sum_{\{j|(i,j) \in \Omega_{BF}\}} \mathcal{P}_{ij,t}^s - P_{i,t}^{g,s} + \mathcal{P}_{L,i,t}^s = 0, \forall i \in \Omega_{NF} \\ d: -\theta_{i,t}^s \leq 0, \quad \forall i \in \Omega_{NF} \\ e: -\mathcal{P}_{L,i,t}^s \leq 0, \quad \forall i \in \Omega_{NF} \end{array} \right\},$$

$$\forall t \in \mathcal{T}_H^s$$

- KKT optimality condition:

Primal feasibility

$$-(1 - w_{ij,t}^{o,s}) M_3 \leq \mathcal{P}_{ij,t}^{s,*} - S_0 B'_{ij} (\theta_{i,t}^{s,*} - \theta_{j,t}^{s,*}) \leq (1 - w_{ij,t}^{o,s}) M_3, \quad \forall (i,j) \in \Omega_{BF}, t \in \mathcal{T}_H^s$$

$$-w_{ij,t}^{o,s} M_3 \leq \mathcal{P}_{ij,t}^{s,*} \leq w_{ij,t}^{o,s} M_3, \quad \forall (i,j) \in \Omega_{BF}, t \in \mathcal{T}_H^s$$

$$\sum_{\{j|(i,j) \in \Omega_{BF}\}} \mathcal{P}_{ij,t}^{s,*} - P_{i,t}^{g,s} + \mathcal{P}_{L,i,t}^{s,*} = 0, \quad \forall i \in \Omega_{NF}, t \in \mathcal{T}_H^s$$

Stationarity

$$\partial \mathcal{L} / \partial \mathcal{P}_{L,i,t}^{s,*} : \alpha_L \mathcal{P}_{L,i,t}^{s,*} + \lambda_{c,i,t}^s - \mu_{e,i,t}^s = 0, \quad \forall i \in \Omega_{NF}, t \in \mathcal{T}_H^s$$

$$\partial \mathcal{L} / \partial \mathcal{P}_{ij,t}^{s,*} : -\lambda_{a,ij,t}^s + \lambda_{b,ij,t}^s + \lambda_{c,i,t}^s - \lambda_{c,j,t}^s = 0, \quad \forall (i,j) \in \Omega_{BF}, t \in \mathcal{T}_H^s$$

$$\partial \mathcal{L} / \partial \theta_{i,t}^{s,*} : \sum_{\{j|(i,j) \in \Omega_{BF}\}} \lambda_{a,ij,t}^s B_{ij} S_0 + 1 - \mu_{d,i,t}^s = 0, \quad \forall i \in \Omega_{NF}, t \in \mathcal{T}_H^s$$

Complementary slackness and dual feasibility

$$0 \leq \mu_{d,i,t}^s \perp \theta_{i,t}^{s,*} \geq 0, \quad \forall i \in \Omega_{NF}, t \in \mathcal{T}_H^s$$

$$0 \leq \mu_{e,i,t}^s \perp \mathcal{P}_{L,i,t}^{s,*} \geq 0, \quad \forall (i,j) \in \Omega_{NF}, t \in \mathcal{T}_H^s$$

On-off line status

$$-(1 - w_{ij,t}^{o,s}) M_4 \leq \lambda_{a,ij,t}^s \leq (1 - w_{ij,t}^{o,s}) M_4, \quad \forall i \in \Omega_{NF}, t \in \mathcal{T}_H^s$$

$$-w_{ij,t}^{o,s} M_4 \leq \lambda_{b,ij,t}^s \leq w_{ij,t}^{o,s} M_4, \quad \forall i \in \Omega_{NF}, t \in \mathcal{T}_H^s$$

Dual Decomposition Algorithm

- A Compact Notation Form of ROD Model

$$z = \min \left\{ \mathbf{c}^\top \mathbf{x} + \sum_{s \in S} p_r(s) \mathbf{q}^\top \mathbf{y}^{R,s} : (\mathbf{x}, \mathbf{y}^{R,s}) \in \mathbf{K}^s, \forall s \in S \right\}$$

where $\mathbf{K}^s = \left\{ (\mathbf{x}, \mathbf{y}^{R,s}) : \mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{T}(s)\mathbf{x} + \mathbf{W}(s)\mathbf{y}^{R,s} = \mathbf{h}(s), \right.$
 $\left. \mathbf{x} \in \{0, 1\}, \mathbf{y}^{R,s} = (\mathbf{y}_B^s, \mathbf{y}_C^s), \mathbf{y}_B^s \in \{0, 1\}, \mathbf{y}_C^s \geq 0 \right\}, \forall s \in S$

- To induce a scenario-based decomposable structure, the copies of the first-stage variables \mathbf{x} are introduced to create the following reformulation

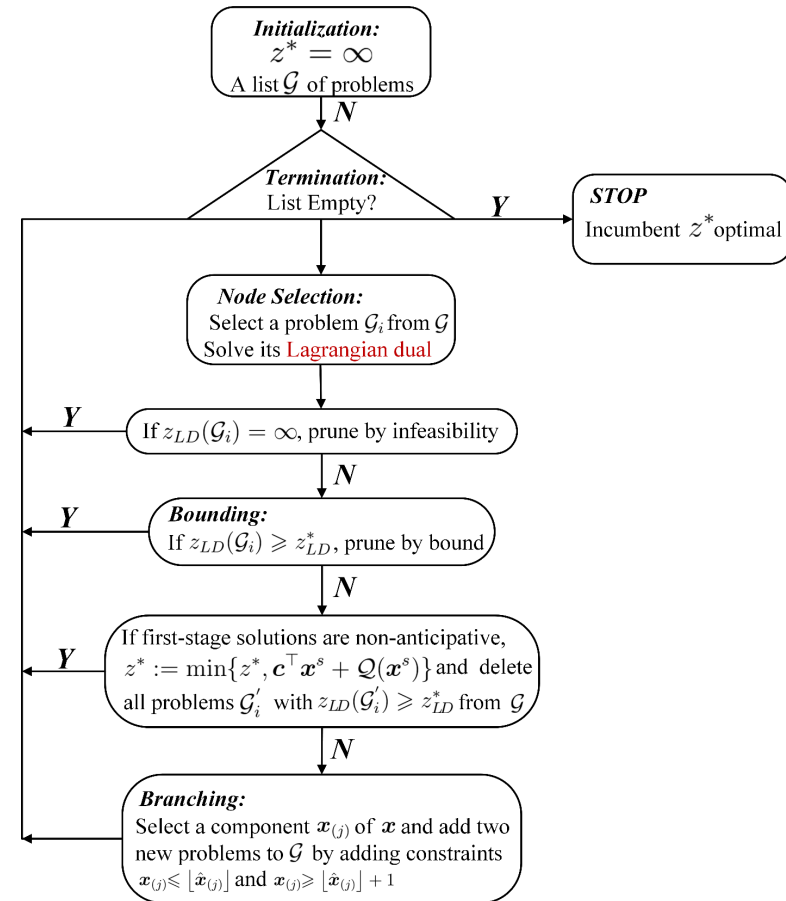
$$z = \min \left\{ \sum_{s \in S} p_r(s) (\mathbf{c}^\top \mathbf{x}^s + \mathbf{q}^\top \mathbf{y}^{R,s}) : \mathbf{x}^1 = \dots = \mathbf{x}^{|S|}, (\mathbf{x}^s, \mathbf{y}^{R,s}) \in \mathbf{K}^s, \forall s \in S \right\}$$

- The Lagrangian relaxation with respect to the nonanticipativity constraint

$$L(\boldsymbol{\mu}) = \sum_{s \in S} L_s(\boldsymbol{\mu}^s) = \sum_{s \in S} \min_{\mathbf{x}^s, \mathbf{y}^{R,s}} \left\{ p_r(s) (\mathbf{c}^\top \mathbf{x}^s + \mathbf{q}^\top \mathbf{y}^{R,s}) + \boldsymbol{\mu}^s \mathbf{x}^s : (\mathbf{x}^s, \mathbf{y}^{R,s}) \in \mathbf{K}^s \right\}$$

- The lower bound of the Lagrangian relaxation:

$$z_{LD} = \max_{\boldsymbol{\mu}} \left\{ \sum_{s \in S} L_s(\boldsymbol{\mu}^s) : \sum_{s \in S} \boldsymbol{\mu}^s = \mathbf{0} \right\}$$



Case Study

TABLE II
THE INVESTMENT COST OF DIFFERENT ROD METHODS

#No.	Methods	Cost(\$)
1	Upgrading pole class	6,000/pole
2	Adding transverse guys to pole	4,000/pole
3	The combination of upgrading and guying pole	10,000/pole
3	Installing a natural gas-fired CHPs as DG with 400kW capacity	1,000/kW
4	Adding an automatic sectionlizer	15,000

*Assume the span of two consecutive poles is 150 ft.

- The IEEE 123-bus system is mapped into a coastal city in Texas.
- The repair cost of a single pole for 6 pole types is assumed to be the same $\chi_{ij,1}^p = \dots = \chi_{ij,6}^p = \4000
- Consider the budget limitation, the total number of backup DGs is limited to be 5
- The total investment cost is \$5,048,000

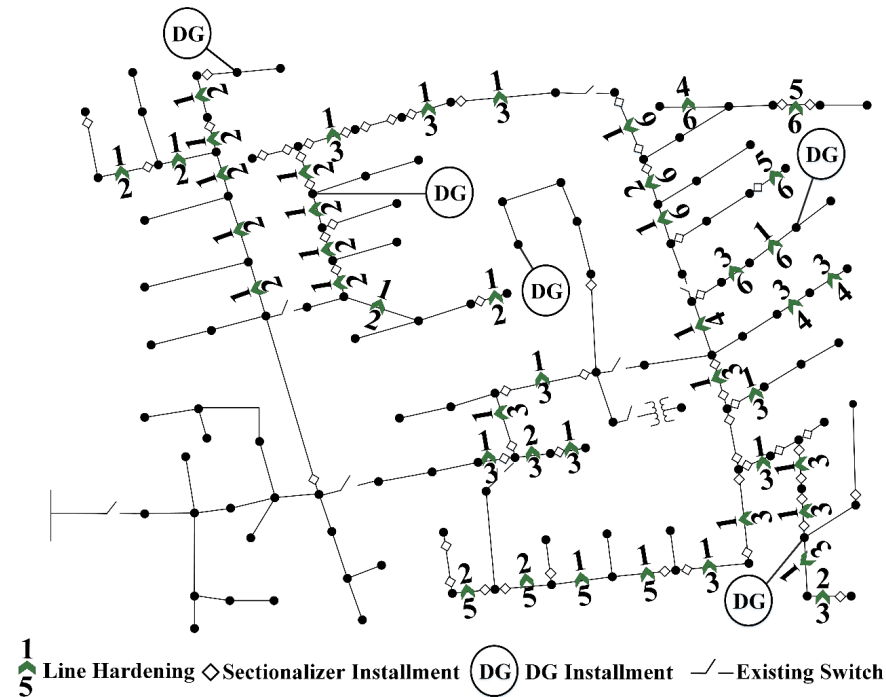
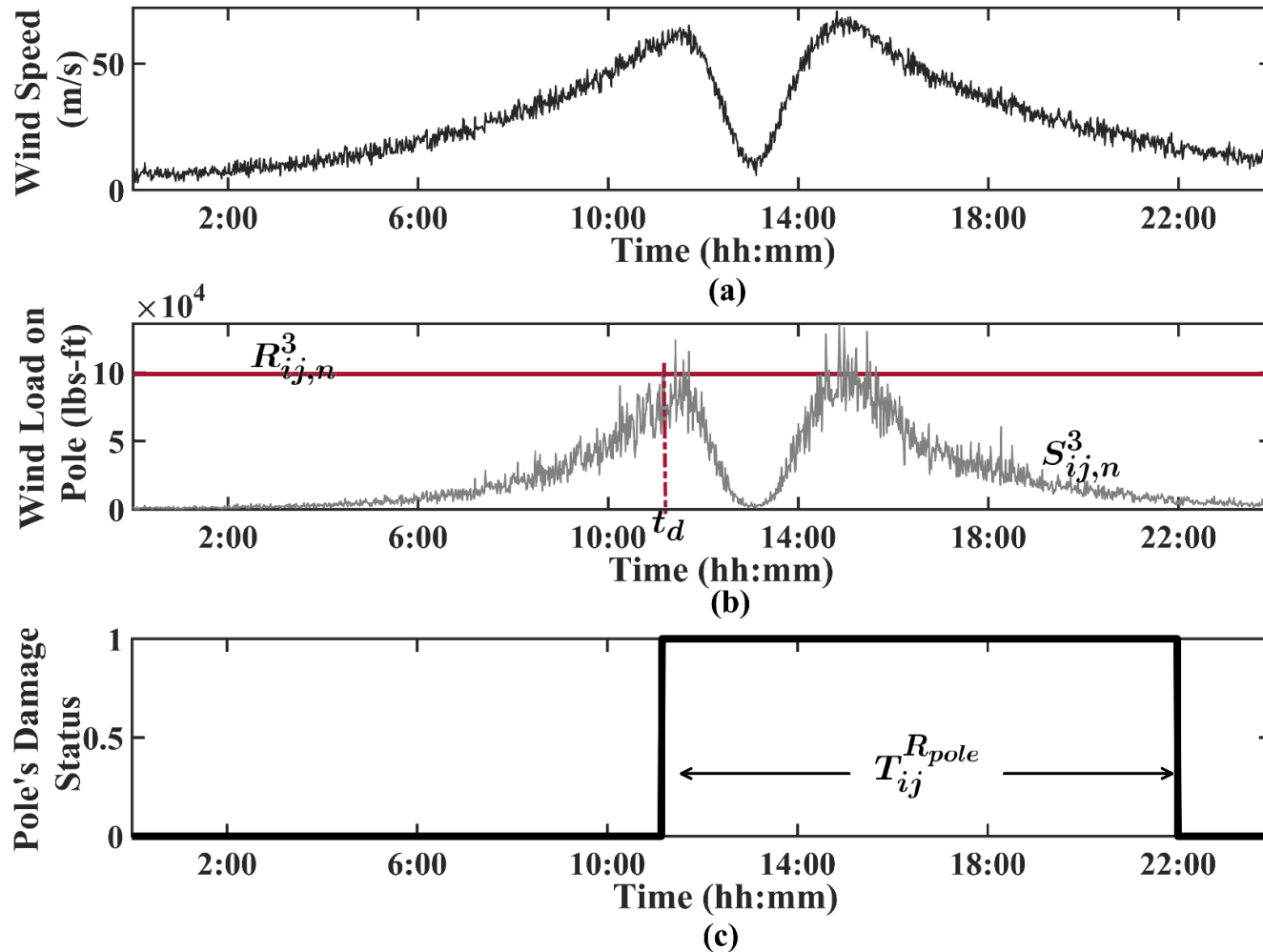


Fig.1. The optimal ROD methods implementation

Simulating A Pole Damage Status in A Hurricane



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Case1: Comparison with and without ROD

- Compare the second stage cost from the hurricane hits the system to the point when all damaged lines are repaired

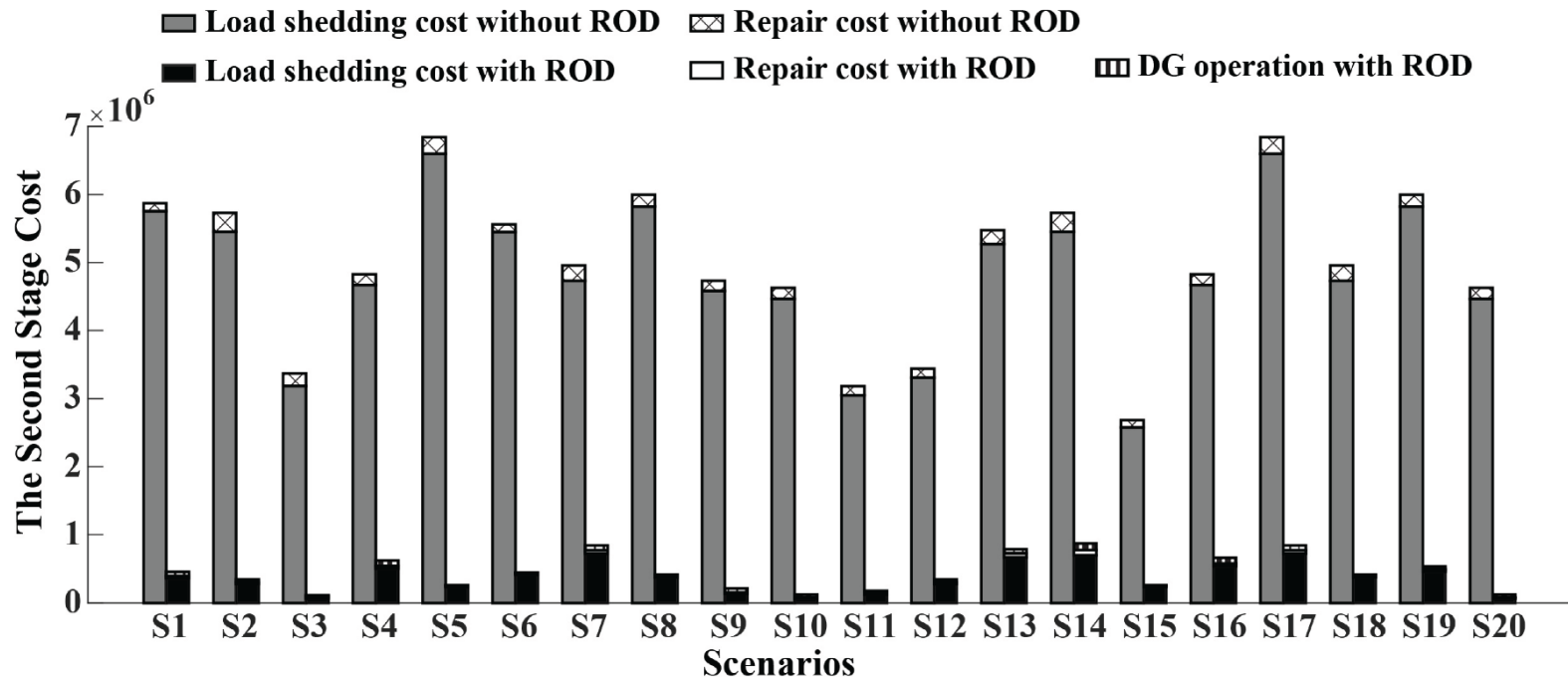


Fig.1. The second stage cost comparison with and without ROD under different scenarios

- The expected second-stage cost with optimal ROD is 8.93% of that without ROD

Case1: Comparison with and without ROD

- Compare the system resilience by the resilience curve, which can be expressed by the percentage of power-served (POPS(t)):

$$\text{POPS}(t) = \sum_{s \in \mathcal{S}} p_r(s) \frac{\sum_{i \in \Omega_N} (1 - y_{i,t}^{r,s}) P_{i,t}^{L,s}}{\sum_{i \in \Omega_N} P_{i,t}^{L,s}}, \forall t \in \mathcal{T}_H$$

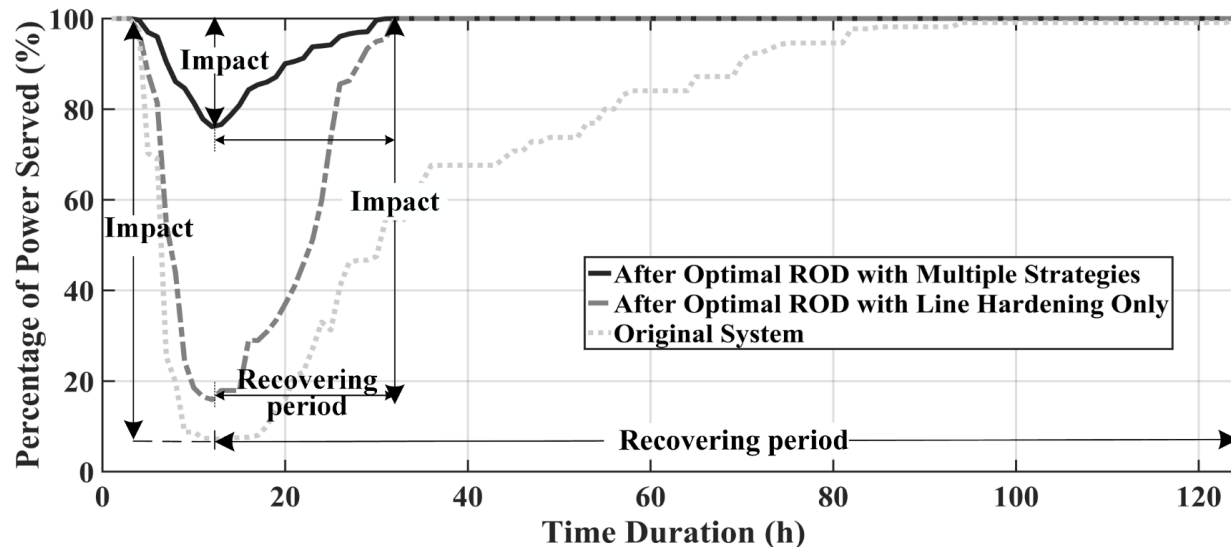


Fig.1. The system resilience curve comparison

- The system with optimal ROD has stronger surviving ability to withstand hurricane and faster recovery
- DGs and automatic sectionalizers can contribute to mitigating the hurricane's impact on the system

Case2: The Self-healing Operation Case

- To validate the novelty of our MILP formulation strategy to solve the challenges of self-healing operation

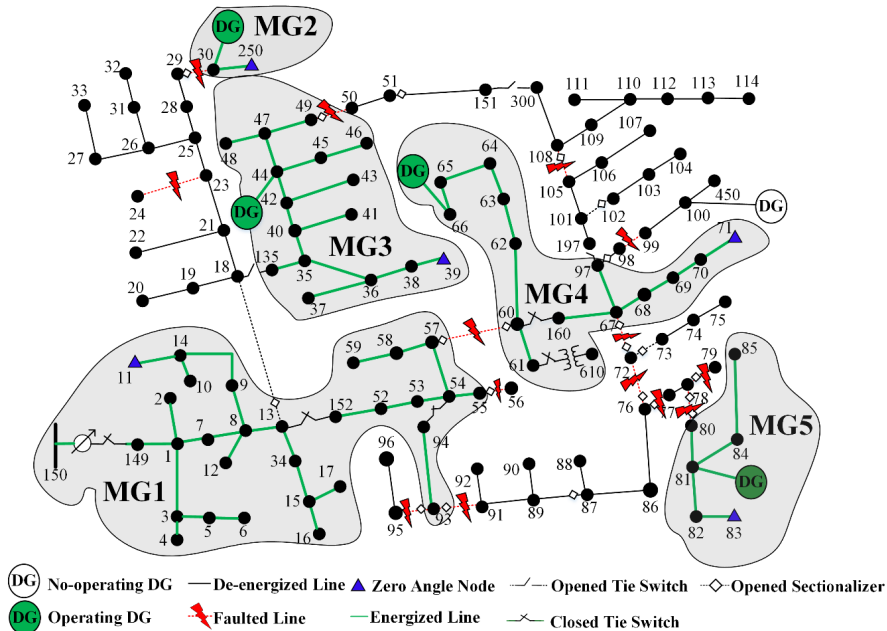


Fig.1. System's self-healing operation at $t = 10$

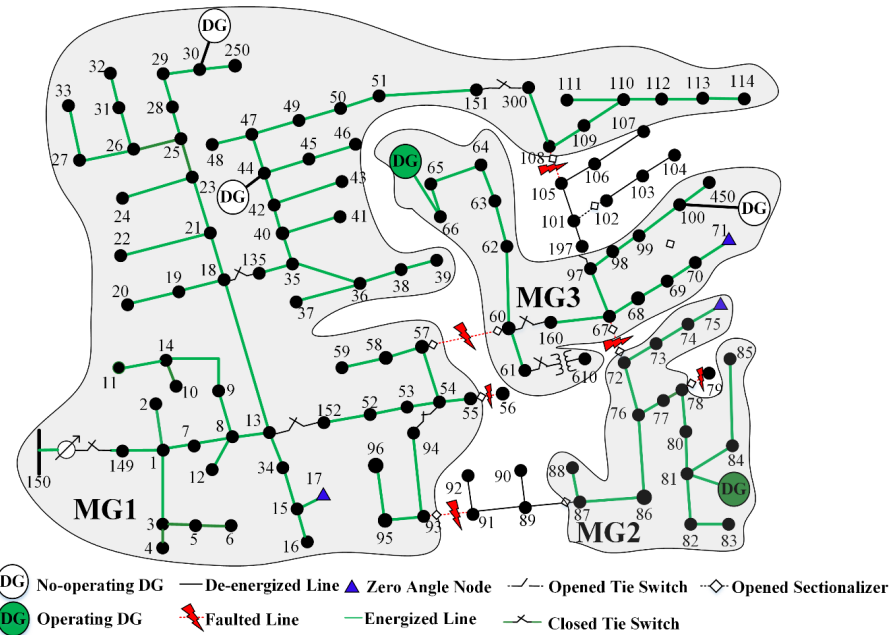
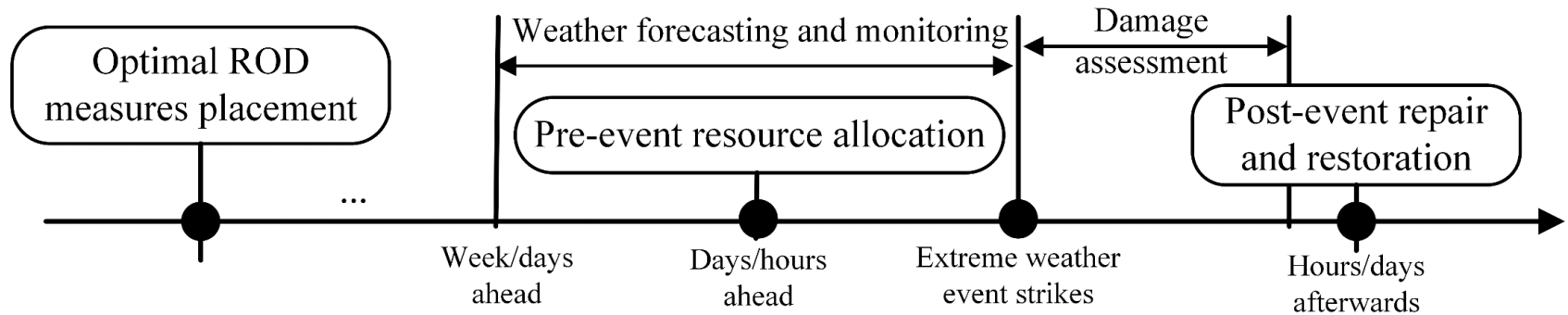


Fig.2. System's self-healing operation at $t = 21$

Conclusions

- A new modeling and solution methodology for resilience-oriented design (ROD) of power distribution systems against wind-induced climatic hazards is proposed
 - The spatial-temporal correlations among ROD decisions, uncertainty space, and system operations during and after extreme weather events are well explored and established
 - A two-stage stochastic mixed-integer model is proposed with the objective to minimize the investment cost in the first-stage and the expected costs of the loss of loads, repairs and DG operations in the second stage.
 - A scenario-based dual composition algorithm is developed to solve the proposed model
 - Numerical studies on the 123-bus distribution system demonstrate the effectiveness of optimal ROD on enhancing the system resilience

Future Work



- We focus on the resource allocation problem ahead of an upcoming extreme weather event
 - consider the uncertainty of the damaged line status, solar irradiance, load demand, and crew repair time
 - explore a tractable measure to model the risk associated with grid components' damages caused by extreme weather events

Publications

Journal Papers

1. **S. Ma**, S. Li, Z. Wang, and F. Qiu, “Resilience-Oriented Distribution System Design with Decision-Dependent Uncertainty,” *IEEE Transactions on Power System*, accepted, 2019.
2. **S. Ma**, Z. Wang, and L. Tesfatsion, “Swing Contracts with Dynamic Reserves for Flexible Service Management,” *IEEE Transactions on Power System*, accepted, 2018
3. **S. Ma**, L. Su, Z. Wang, and F. Qiu, “Resilience Enhancement of Distribution Grids Against Extreme Weather Events,” *IEEE Transactions on Power System*, vol. 33, no. 5, pp. 4842-4853, Sept. 2018.
4. A. Arif, **S. Ma**, Z. Wang, J Wang, S. M. Ryan, and C. Chen, “Optimizing Service Restoration in Distribution Systems With Uncertain Repair Time and Demand,” *IEEE Transactions on Power System*, vol. 33, no. 6, pp. 6828-6838, Nov. 2018.
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Conference Paper

1. **S. Ma**, N. Carrington, A. Arif, and Z. Wang, “Resilience Assessment of Self-healing Distribution Systems Under Extreme Weather Events”, *IEEE PES General Meeting*, accepted, 2019.
2. **S. Ma**, S. Li, Z. Wang, A. Arif, and K. Ma., “A novel MILP formulation for fault isolation and network reconfiguration in active distribution systems,” *IEEE PES General Meeting*, Portland, OR, 2018, pp. 1-5.
3. A. Arif, **S. Ma**, and Z. Wang, “Dynamic reconfiguration and fault isolation for a selfhealing distribution system,” *IEEE PES Transm. Distrib. Conf. Expo.*, Denver, CO, 2018, pp. 1-5.73
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Thank You!
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